

Likely chaotic transitions of large-scale fluid flows using a stochastic transport model

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Likely chaotic transitions of large-scale fluid flows using a stochastic transport model

Valentin Resseguier,
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Motivations for deriving random fluid dynamics models

- Rigorously identified subgrid dynamics effects
- Injecting likely small-scale dynamics
- Quantification of modeling errors:
 especially for Data assimilation: ensemble forecasts
- **Predicting possible distinct scenarios**

Contents

- Chaotic transitions
- Dynamics under location uncertainty
- Ensemble of simulations

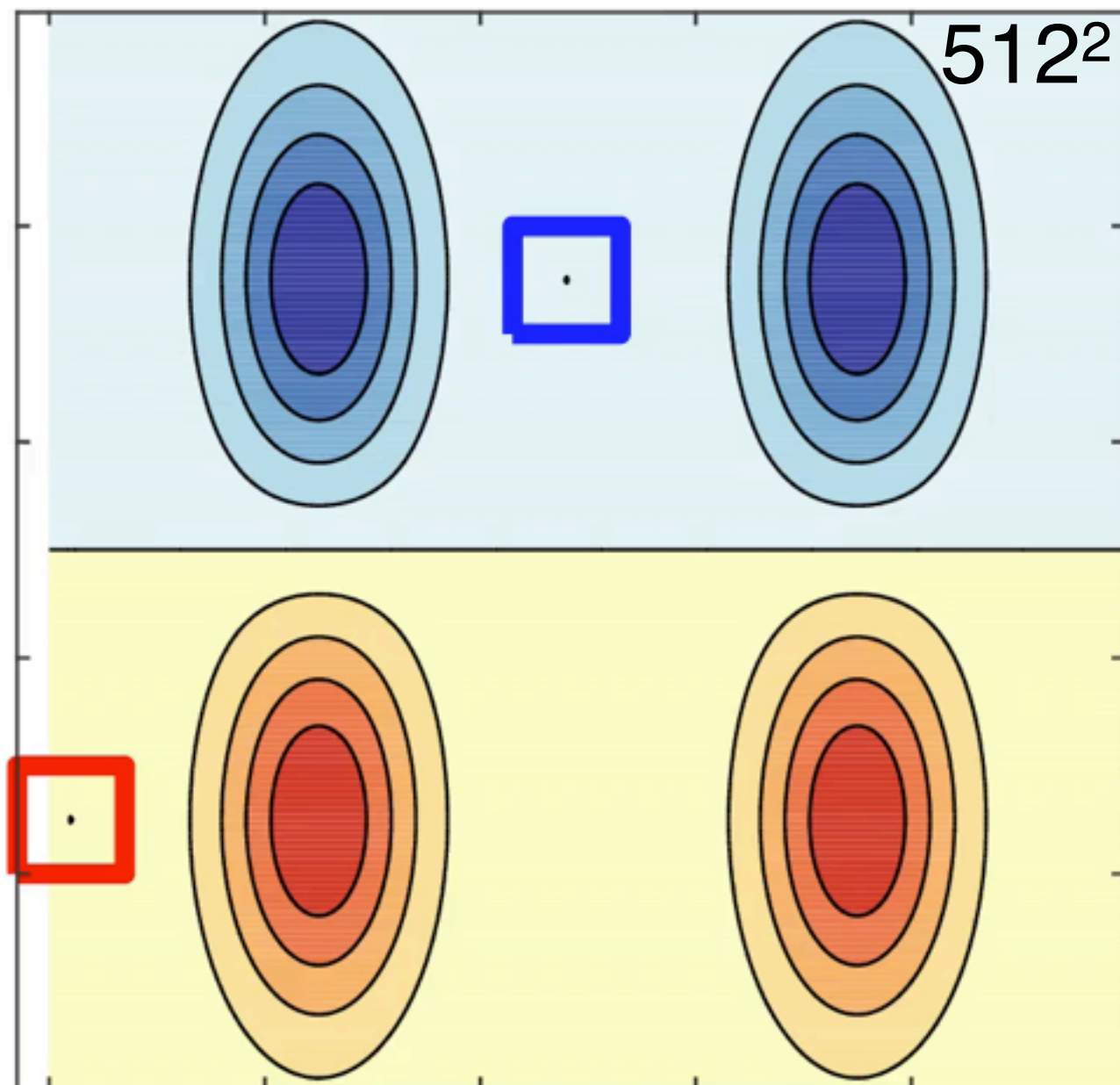
Chaotic transitions

in SQG dynamics

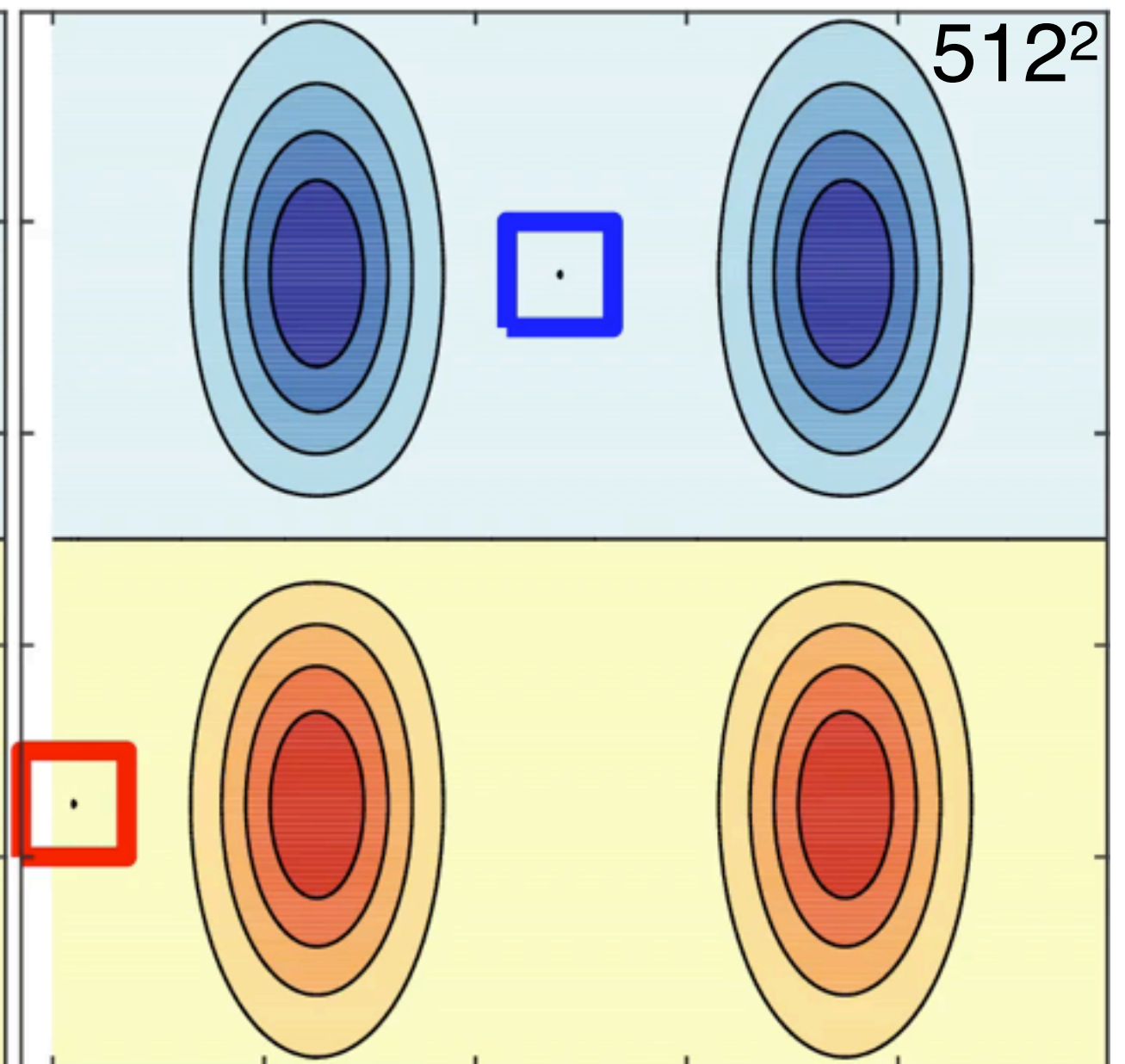
Reference flow: deterministic SQG 512^2

Initial condition 1  Scenario 1

$t = 0$ days



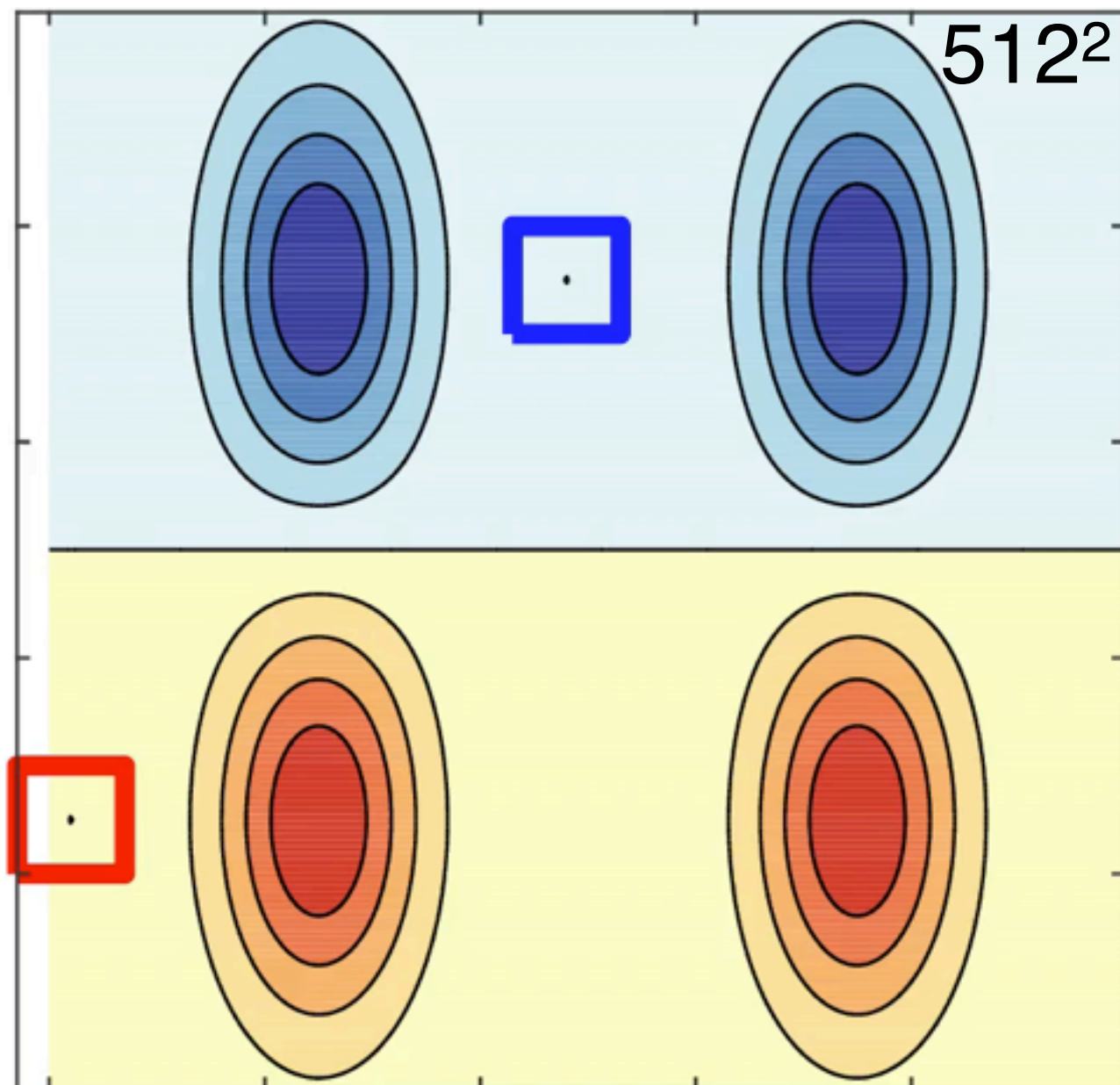
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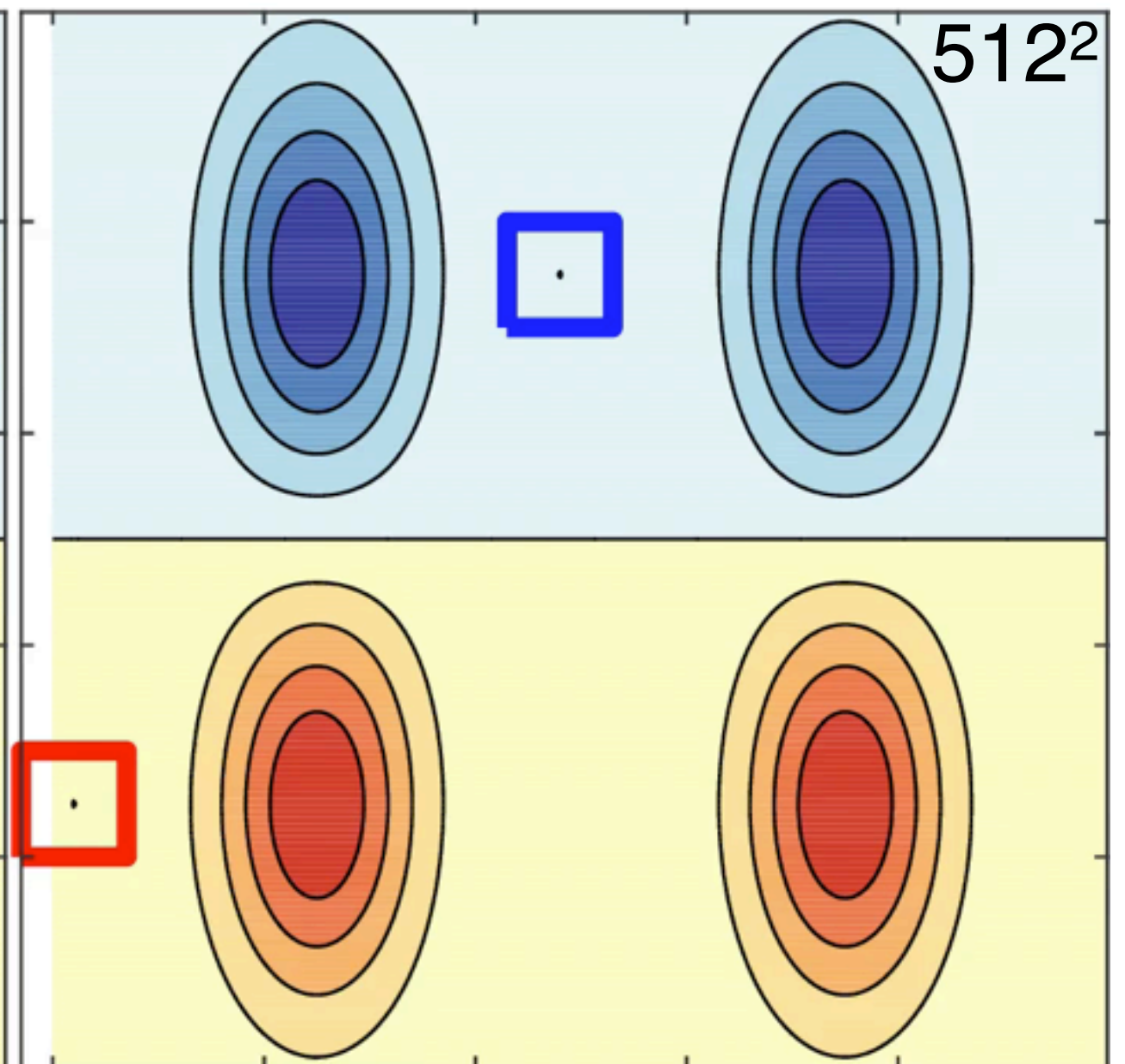
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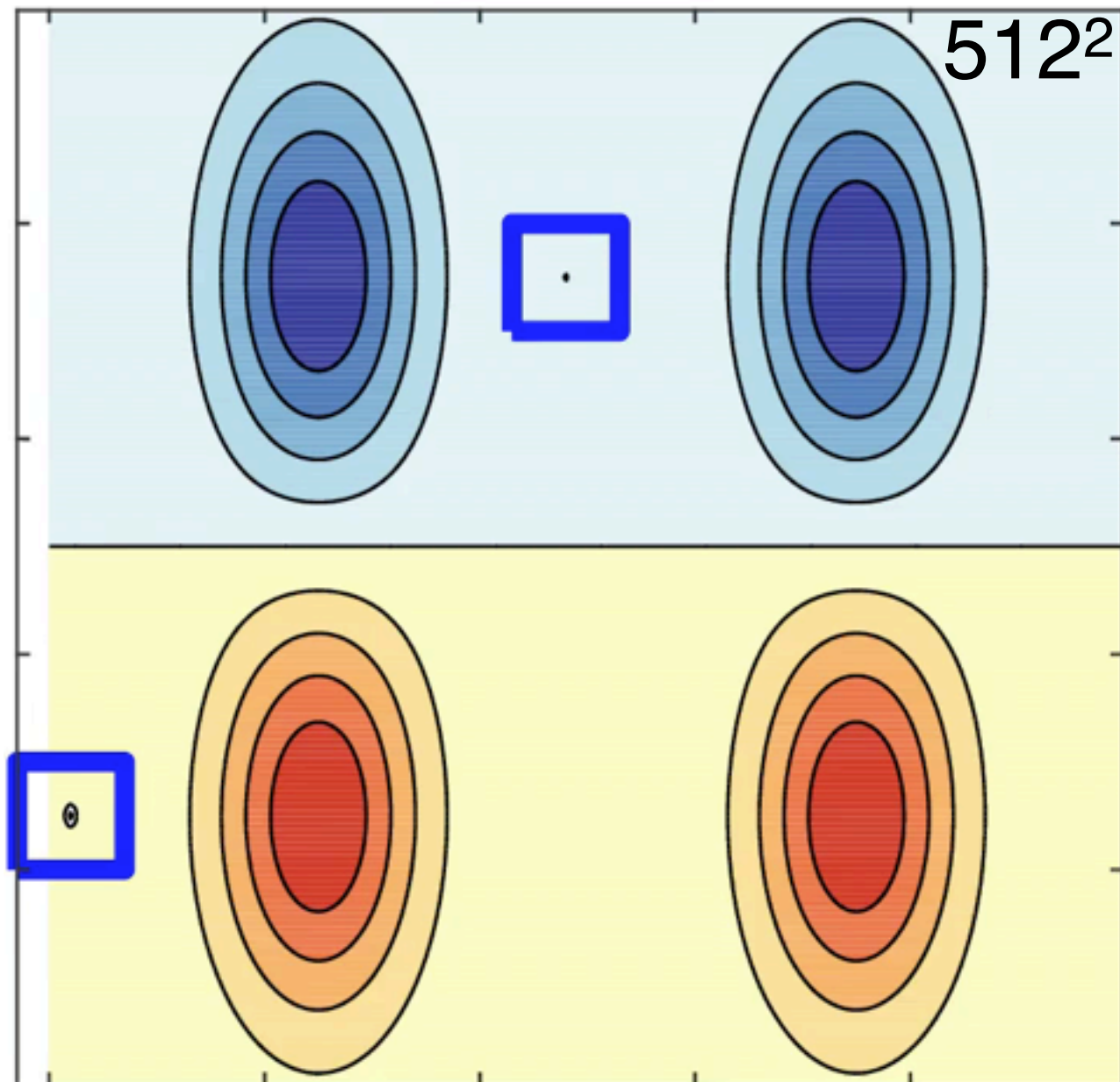
$t = 0$ days



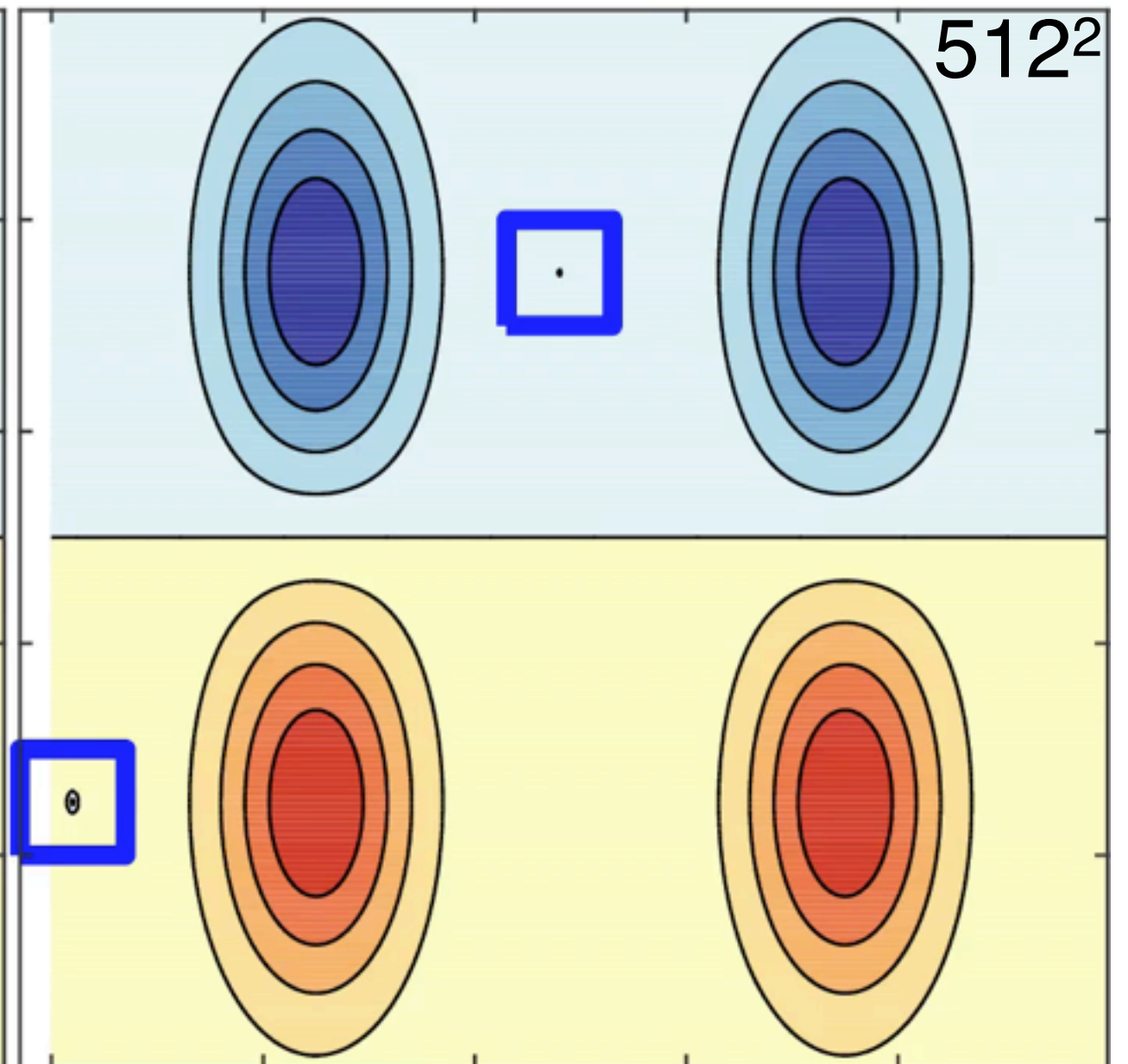
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Initial condition 2  Scenario 2

$t = 0$ days



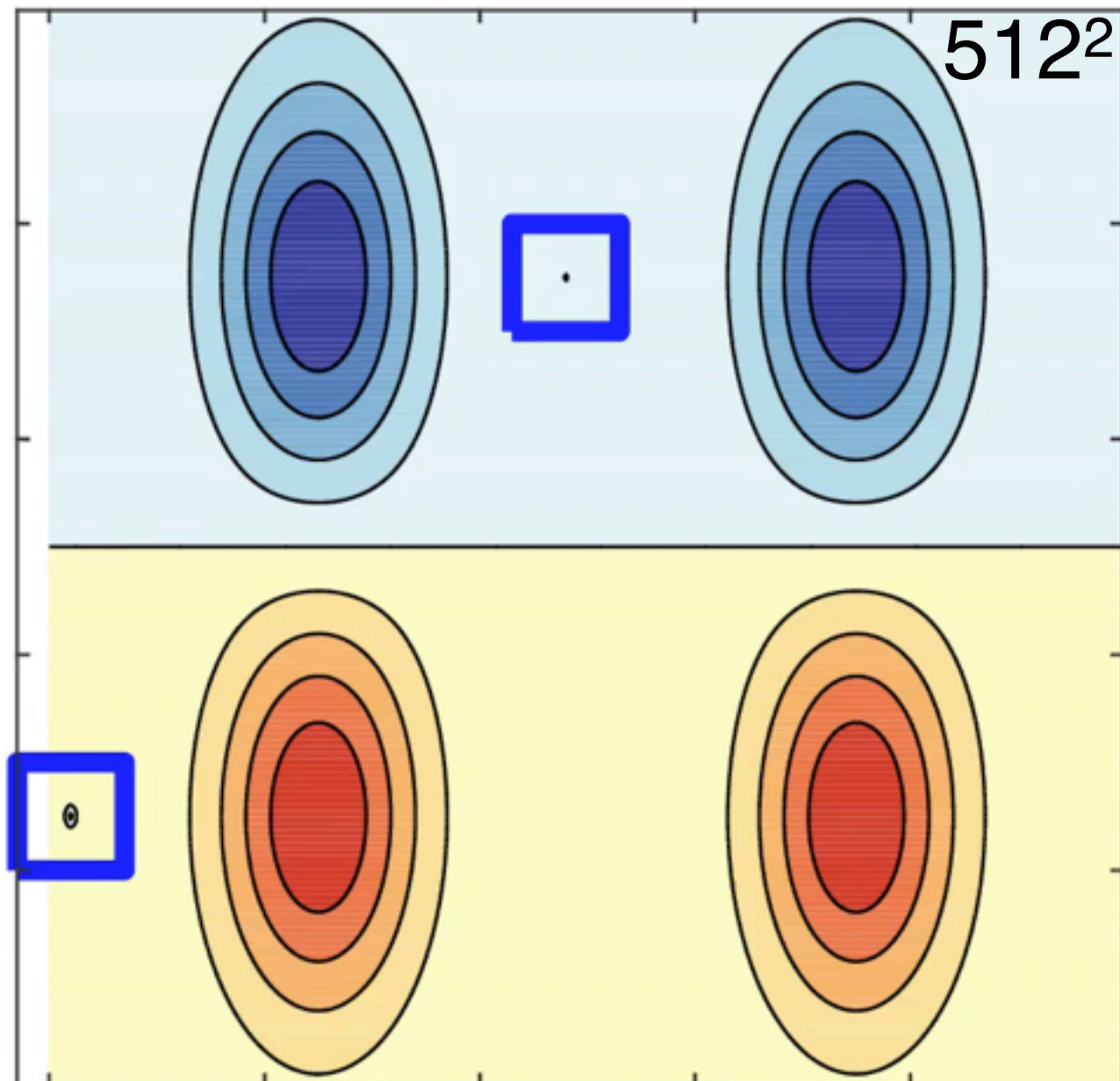
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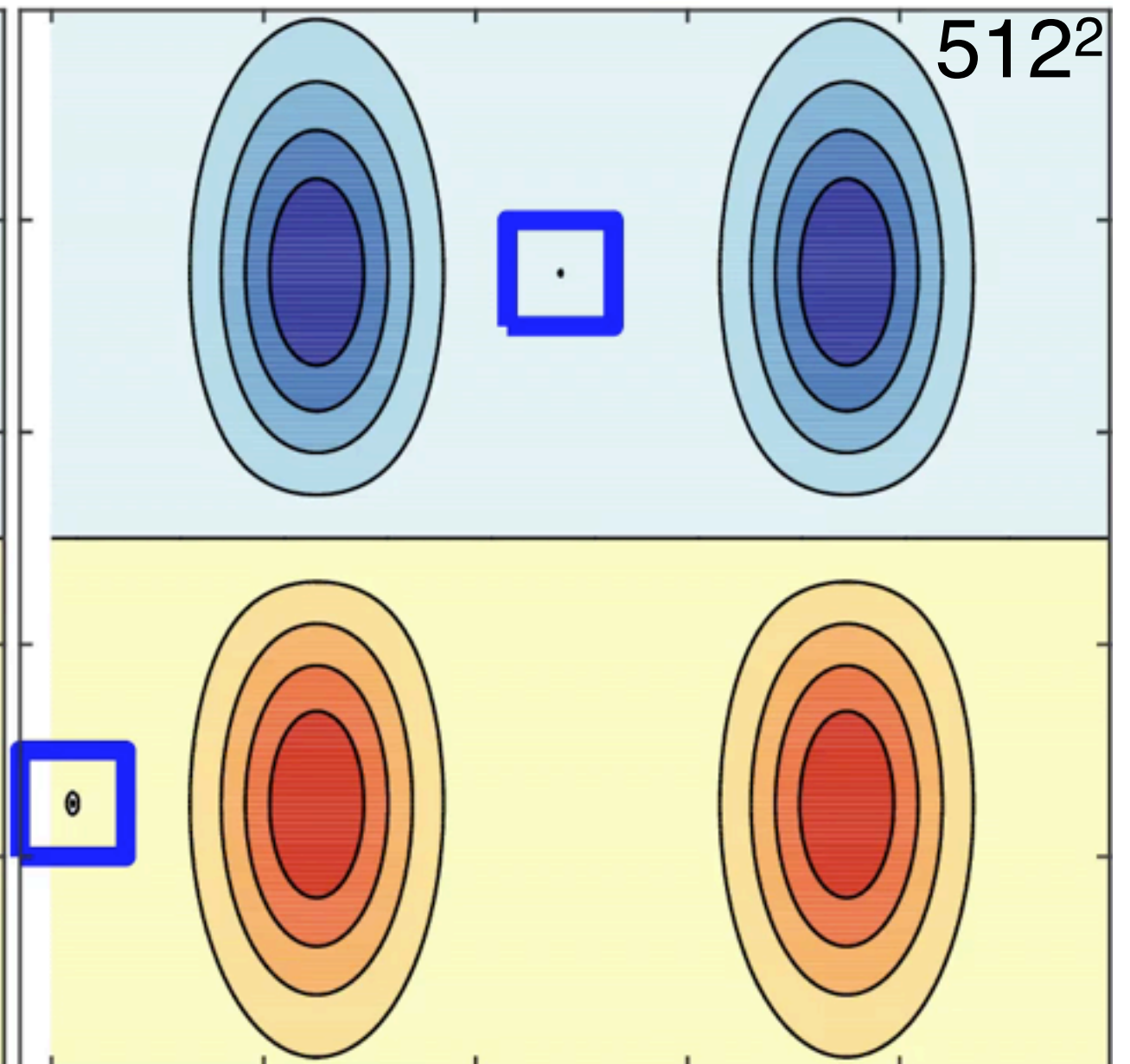
Reference flow: deterministic SQG 512^2

Initial condition 2  Scenario 2

$t = 0$ days



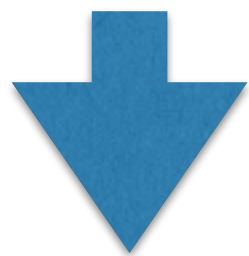
$t = 0$ days



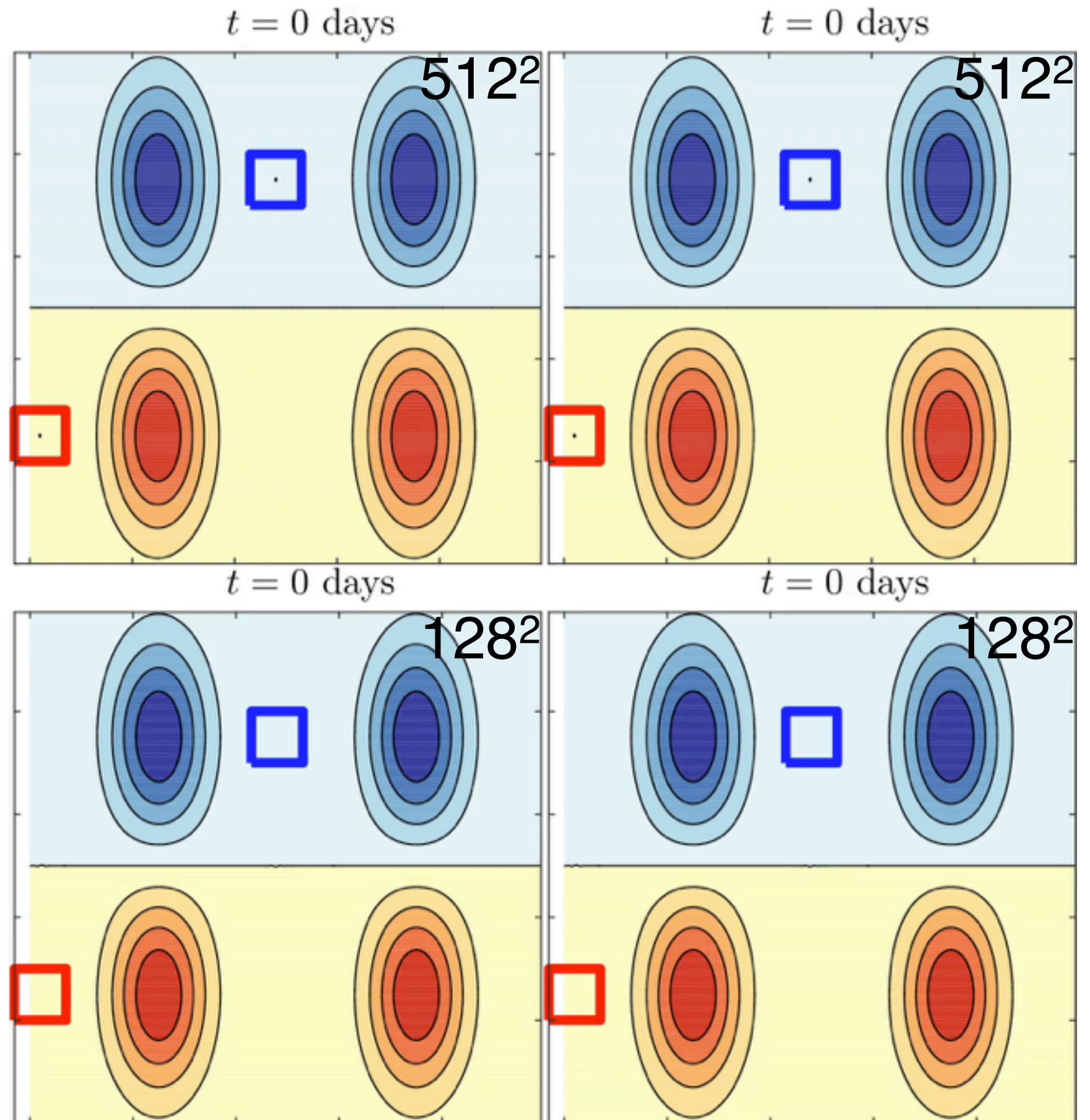
Reference flow:
deterministic
SQG

512² versus 128²

Initial condition 1



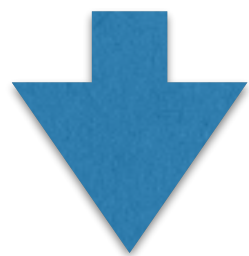
?



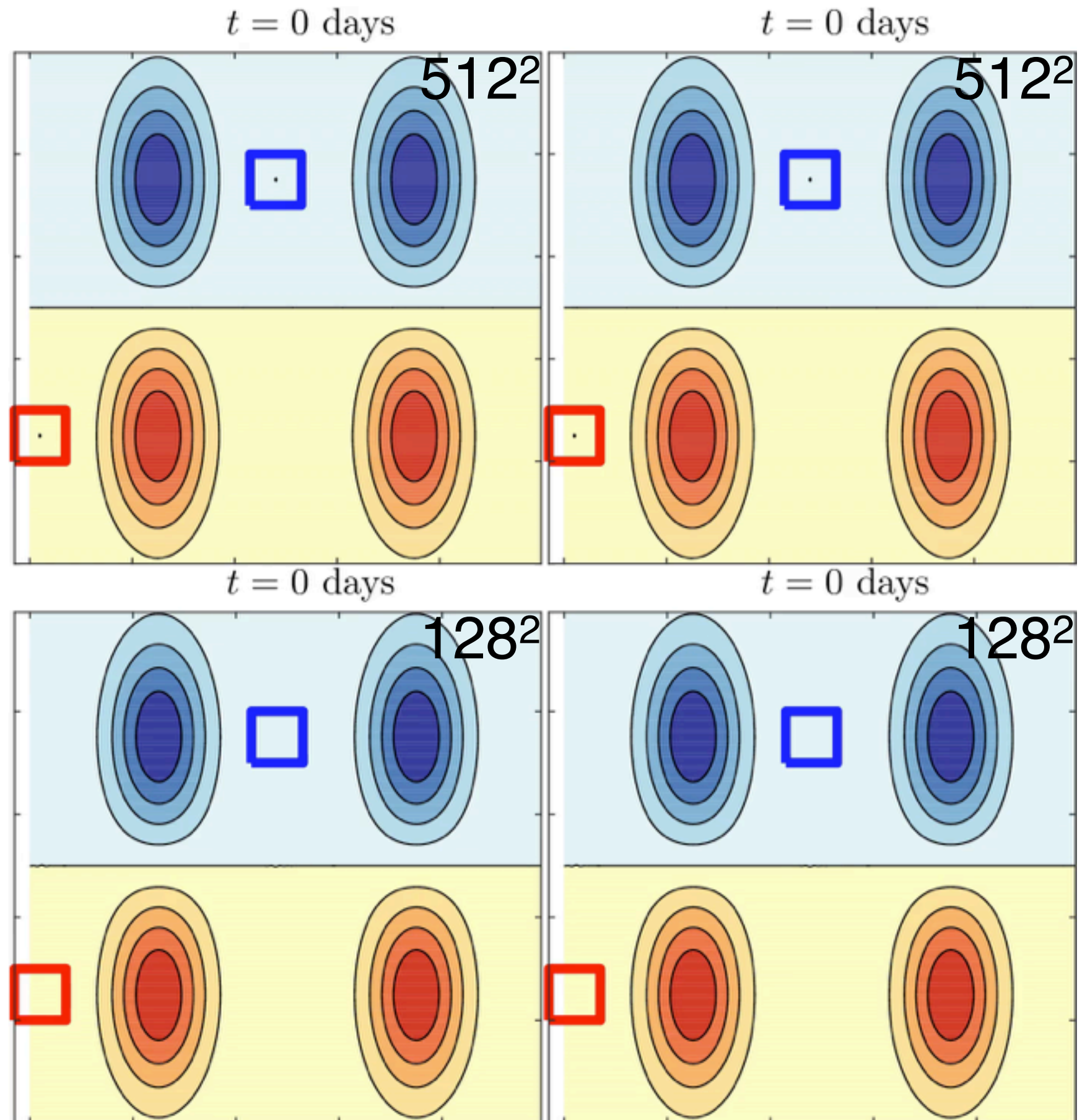
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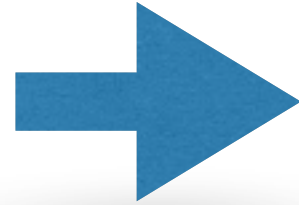
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Dynamics under location uncertainty

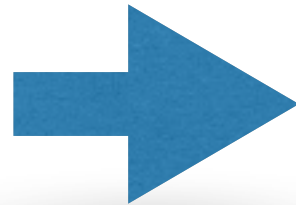
Random equations

- Random initial conditions



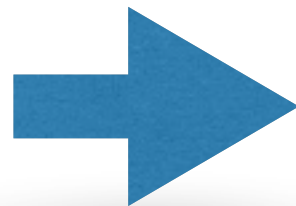
Underdispersive +
need large ensemble

- Arbitrary Gaussian forcing



Adding energy
+ wrong phase

- Averaging, homogenization



Assumptions and
energy issues

- Adding white
random velocity



$$\boldsymbol{v} = \boldsymbol{w} + \sigma \dot{\boldsymbol{B}}$$

Advection of tracer Θ

$$\frac{D\Theta}{Dt} = 0$$

Advection of tracer Θ

Advection of tracer Θ

$$\partial_t \Theta + \boldsymbol{w}^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

Advection of tracer Θ

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta}_{\text{Advection}} = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

Advection of tracer Θ

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta}_{\text{Advection}} = \underbrace{\nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)}_{\text{Diffusion}}$$

Advection of tracer Θ

The diagram illustrates the advection of a tracer Θ . The equation is presented as:

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta}_{\text{Advection}} = \underbrace{\nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)}_{\text{Diffusion}}$$

An orange arrow points to the w^* term in the advection part, labeled "Drift correction".

Advection of tracer Θ

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta}_{\text{Advection}} = \underbrace{\nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)}_{\text{Diffusion}}$$

Drift correction

Multiplicative random forcing

Advection of tracer Θ

The diagram illustrates the advection of a tracer Θ . The equation is presented as:

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta}_{\text{Advection}} = \underbrace{\nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)}_{\text{Diffusion}}$$

The advection term is enclosed in a light blue box labeled "Advection". Within this box, the term w^* is circled in orange, and the term $\sigma \dot{B} \cdot \nabla \Theta$ is circled in purple. An orange arrow points from the text "Drift correction" to the w^* term. A purple arrow points from the text "Multiplicative random forcing" to the $\sigma \dot{B} \cdot \nabla \Theta$ term. The diffusion term is enclosed in a light green box labeled "Diffusion".

Advection of tracer Θ

$$\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

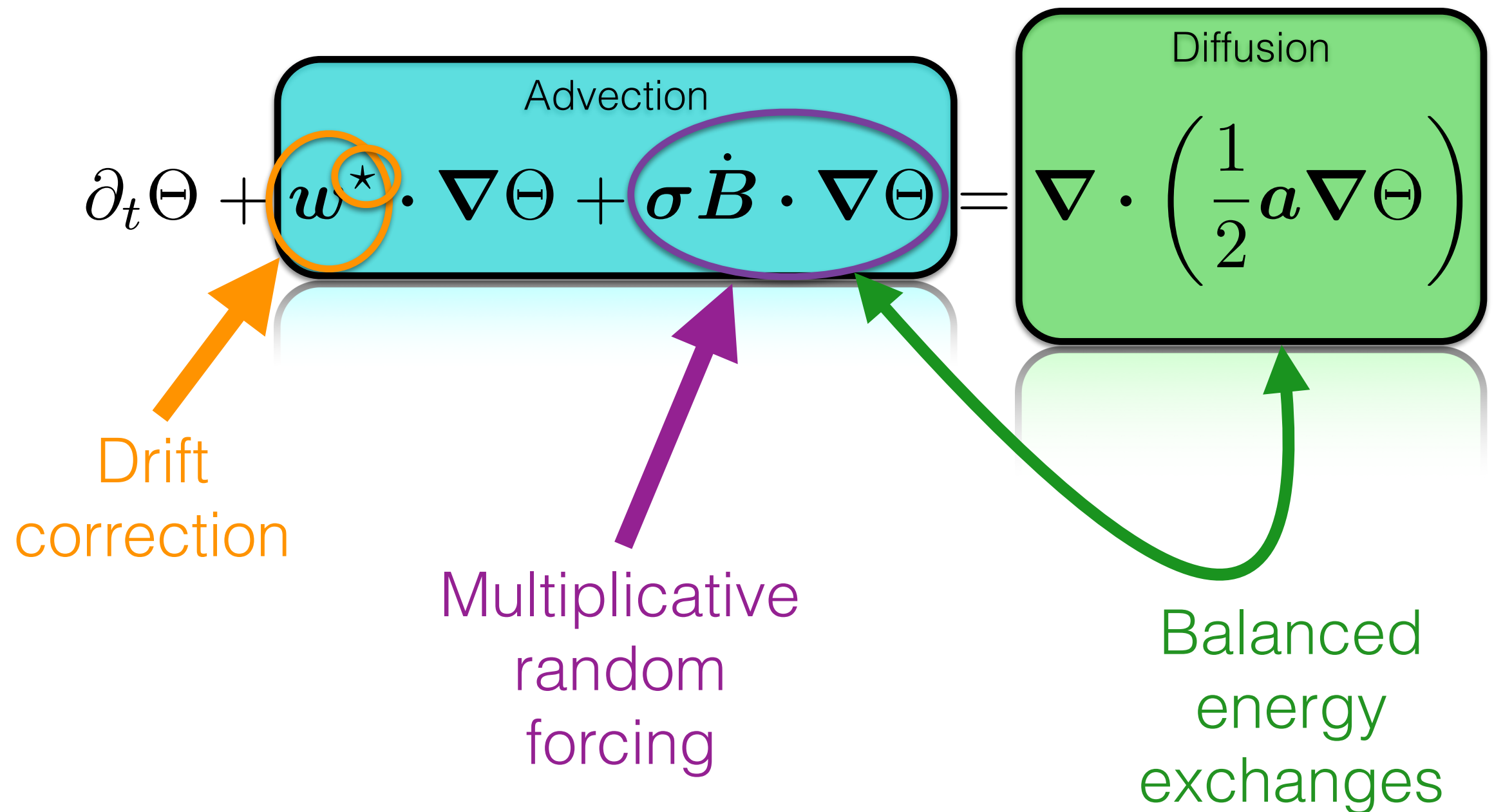
Advection

Diffusion

Drift correction

Multiplicative random forcing

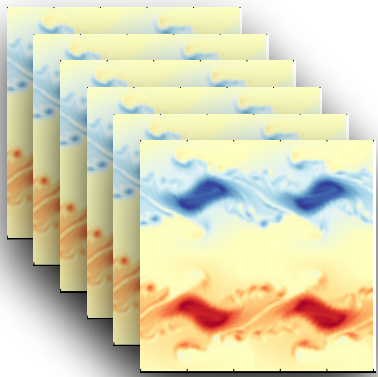
Advection of tracer Θ



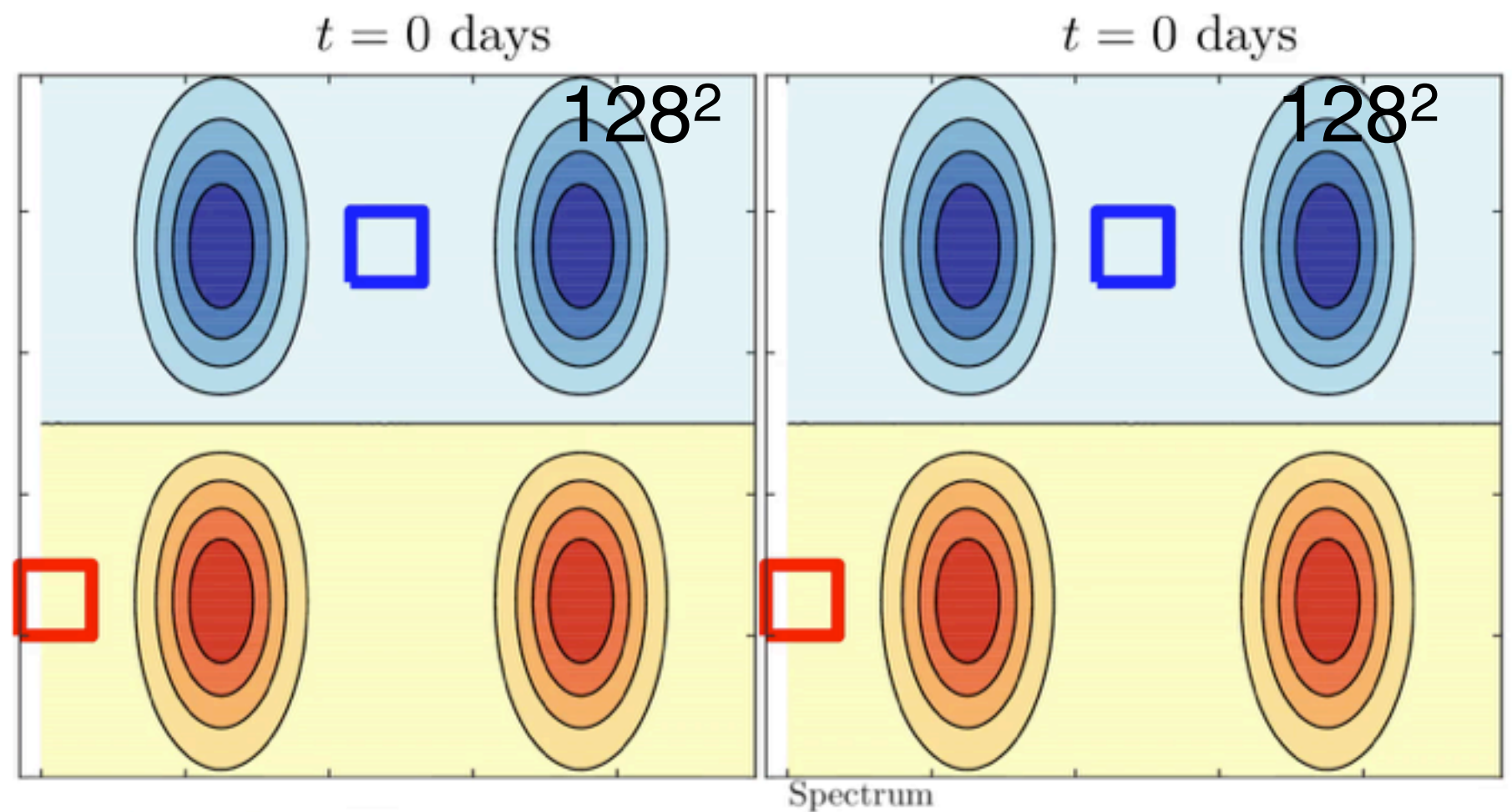
Ensemble of simulations

with SQG MU

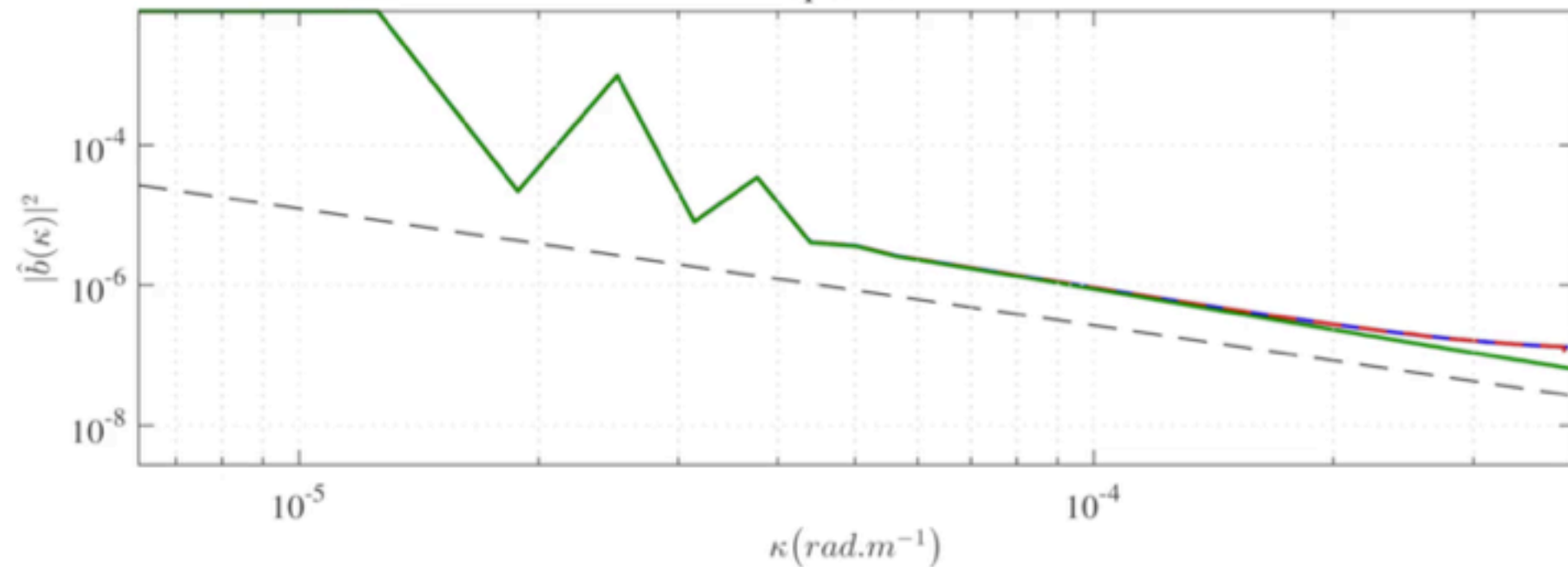
Code available online

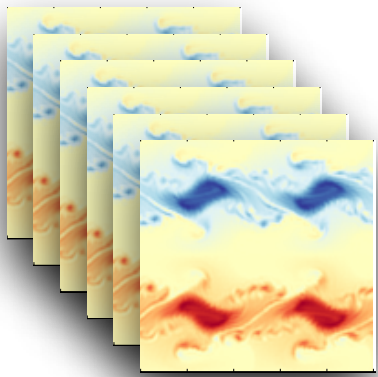


examples of
realizations:

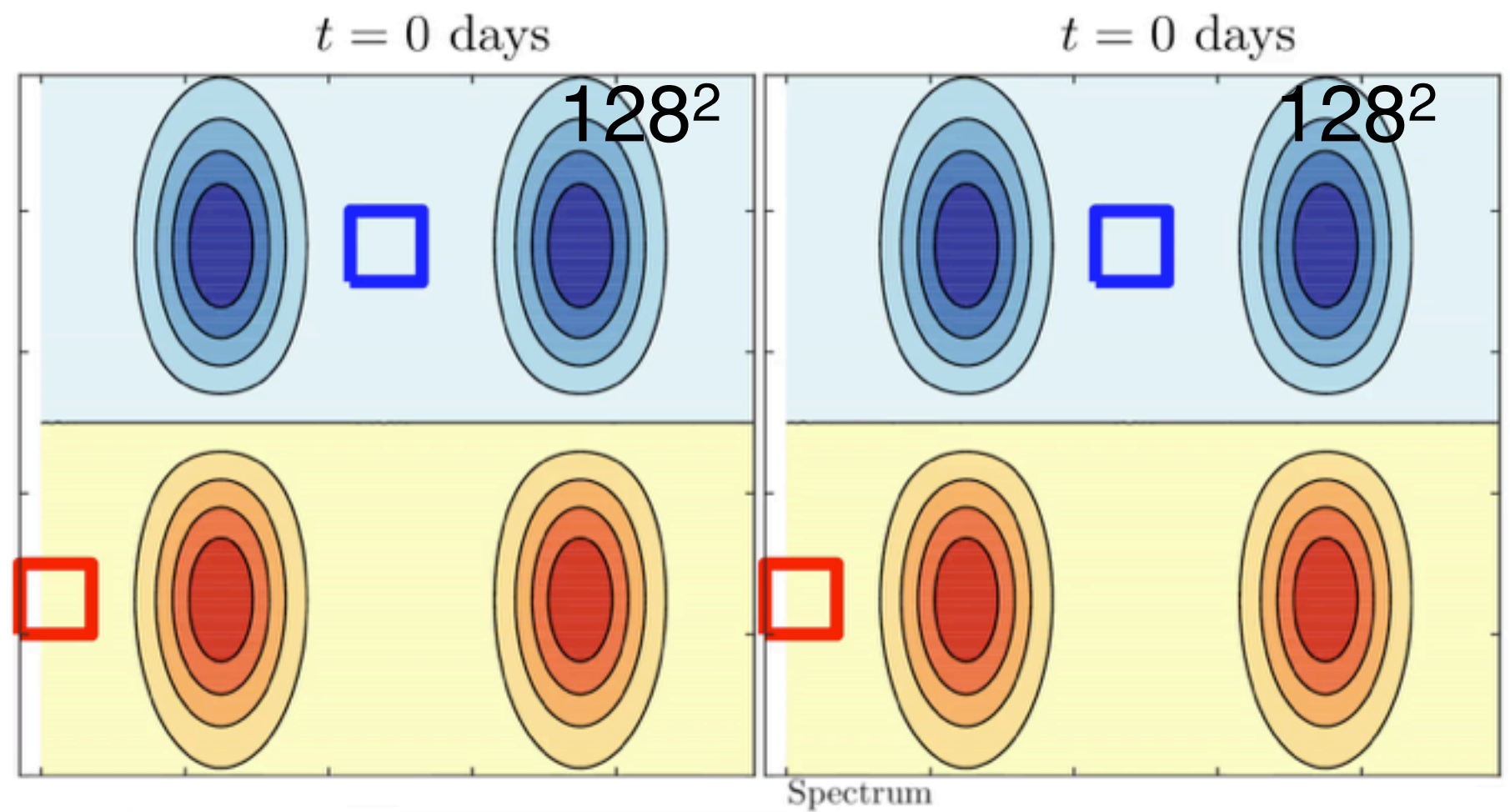


Spectrum
mean²/variance:

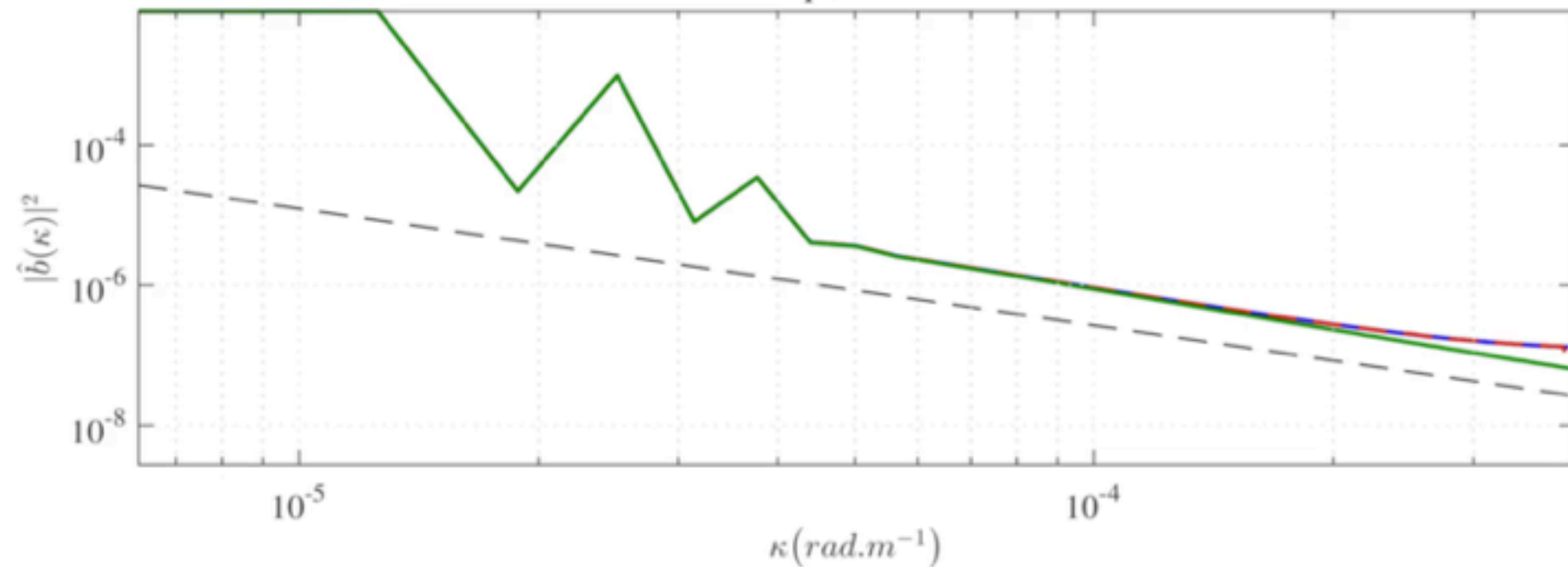


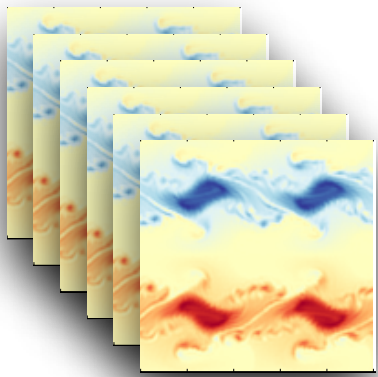


examples of
realizations:



Spectrum
mean²/variance:



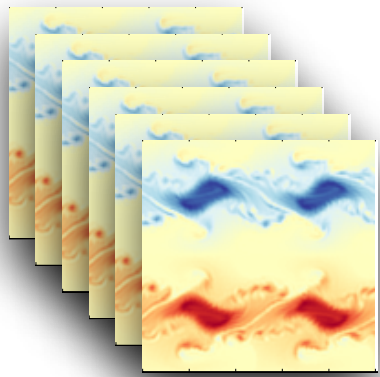


Visualization of the ensemble

At a fixed time t ,

Principal Component Analysis (EOF)

$$\Theta^{(i)}(\mathbf{x}, t) \approx \hat{\mathbb{E}}(\Theta)(\mathbf{x}, t) + \sum_{n=1}^{N_{EOF}} c_n^{(i)}(t) \Psi_n(\mathbf{x}, t)$$



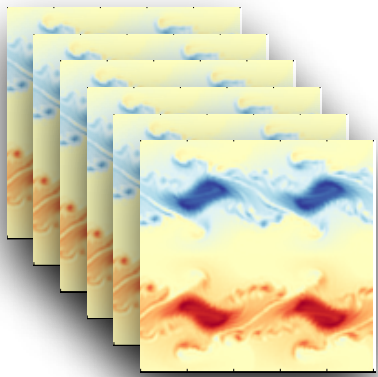
Visualization of the ensemble

At a fixed time t ,

Principal Component Analysis (EOF)

$$\Theta^{(i)}(\mathbf{x}, t) \approx \hat{\mathbb{E}}(\Theta)(\mathbf{x}, t) + \sum_{n=1}^{N_{EOF}} c_n^{(i)}(t) \Psi_n(\mathbf{x}, t)$$

i^{th} realization



Visualization of the ensemble

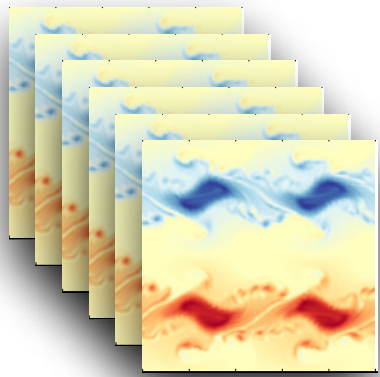
At a fixed time t ,

Principal Component Analysis (EOF)

$$\Theta^{(i)}(\mathbf{x}, t) \approx \hat{\mathbb{E}}(\Theta)(\mathbf{x}, t) + \sum_{n=1}^{N_{EOF}} c_n^{(i)}(t) \Psi_n(\mathbf{x}, t)$$

i^{th} realization

Mean



Visualization of the ensemble

At a fixed time t ,

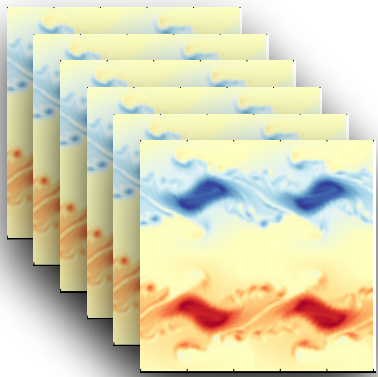
Principal Component Analysis (EOF)

$$\Theta^{(i)}(\mathbf{x}, t) \approx \hat{\mathbb{E}}(\Theta)(\mathbf{x}, t) + \sum_{n=1}^{N_{EOF}} c_n^{(i)}(t) \Psi_n(\mathbf{x}, t)$$

i^{th} realization

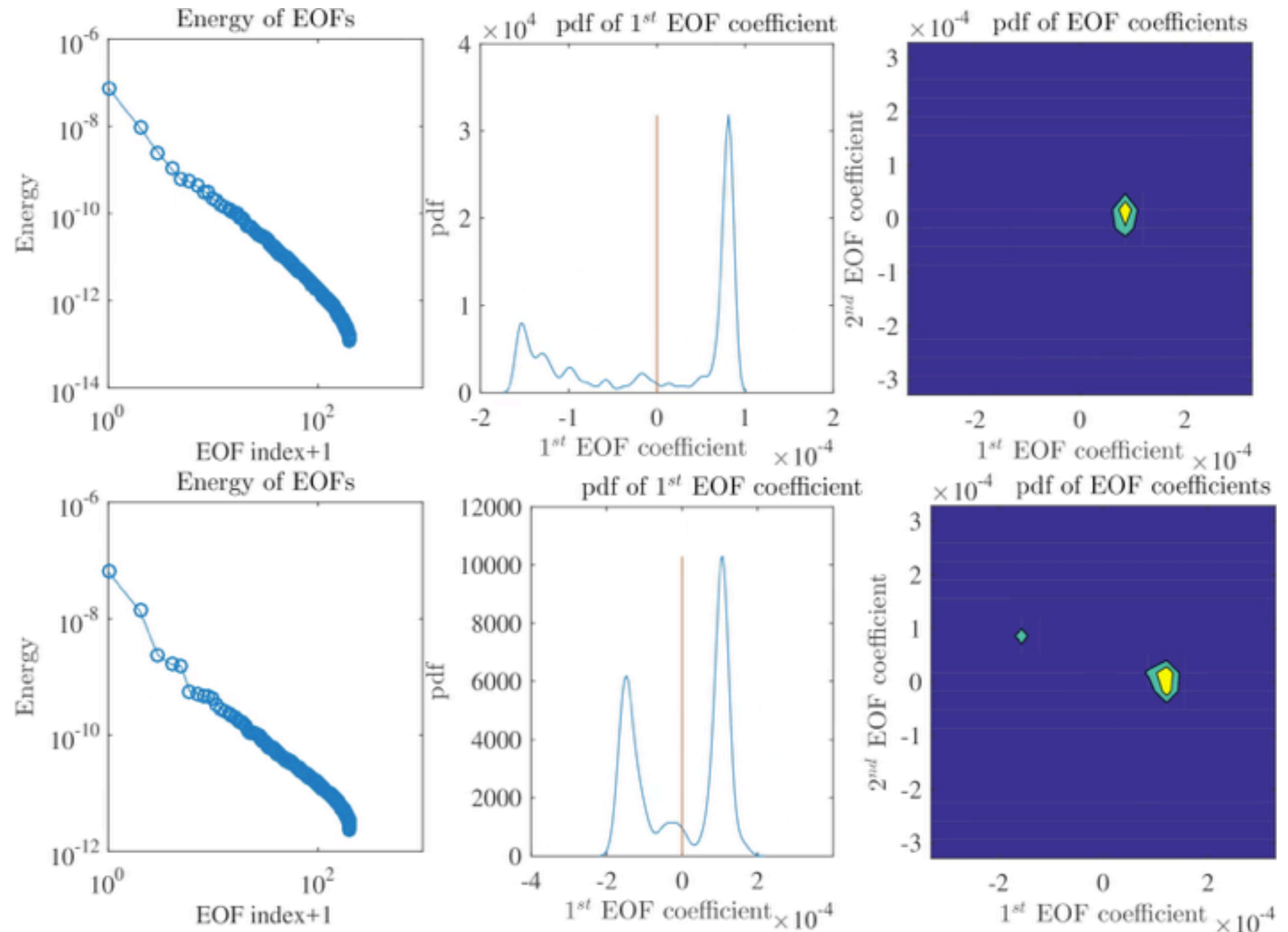
Mean

(random) EOF coefficient

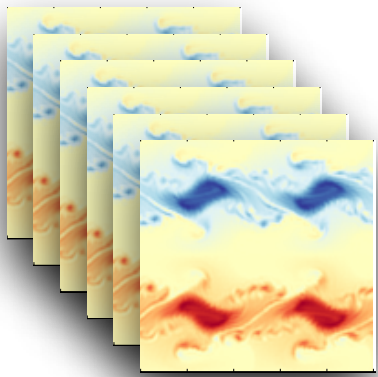


Visualization of the ensemble (200 realizations)

Random
initial
conditions

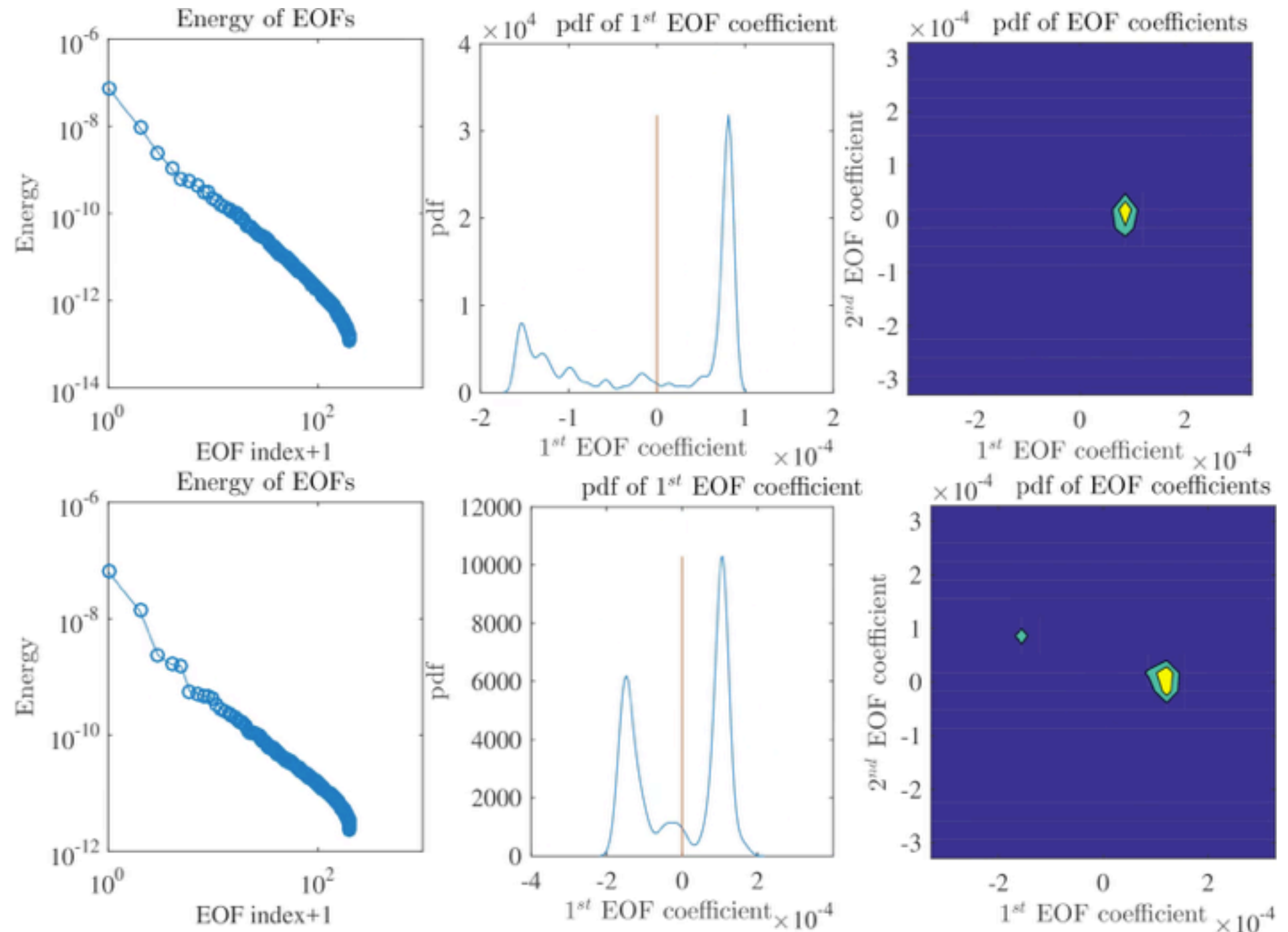


Under location
uncertainty

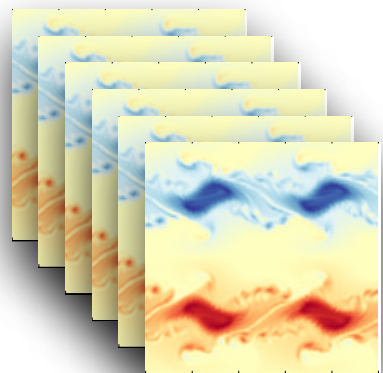


Visualization of the ensemble (200 realizations)

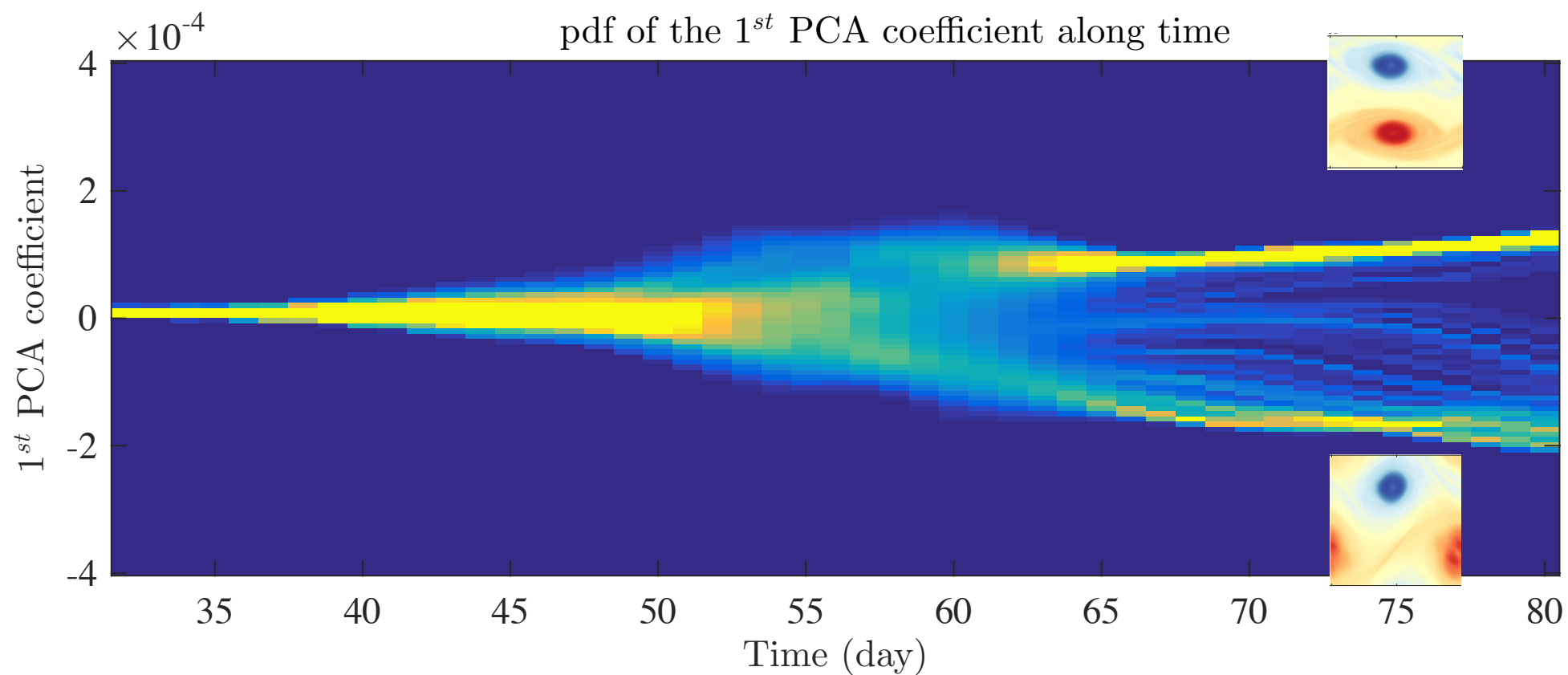
Random
initial
conditions



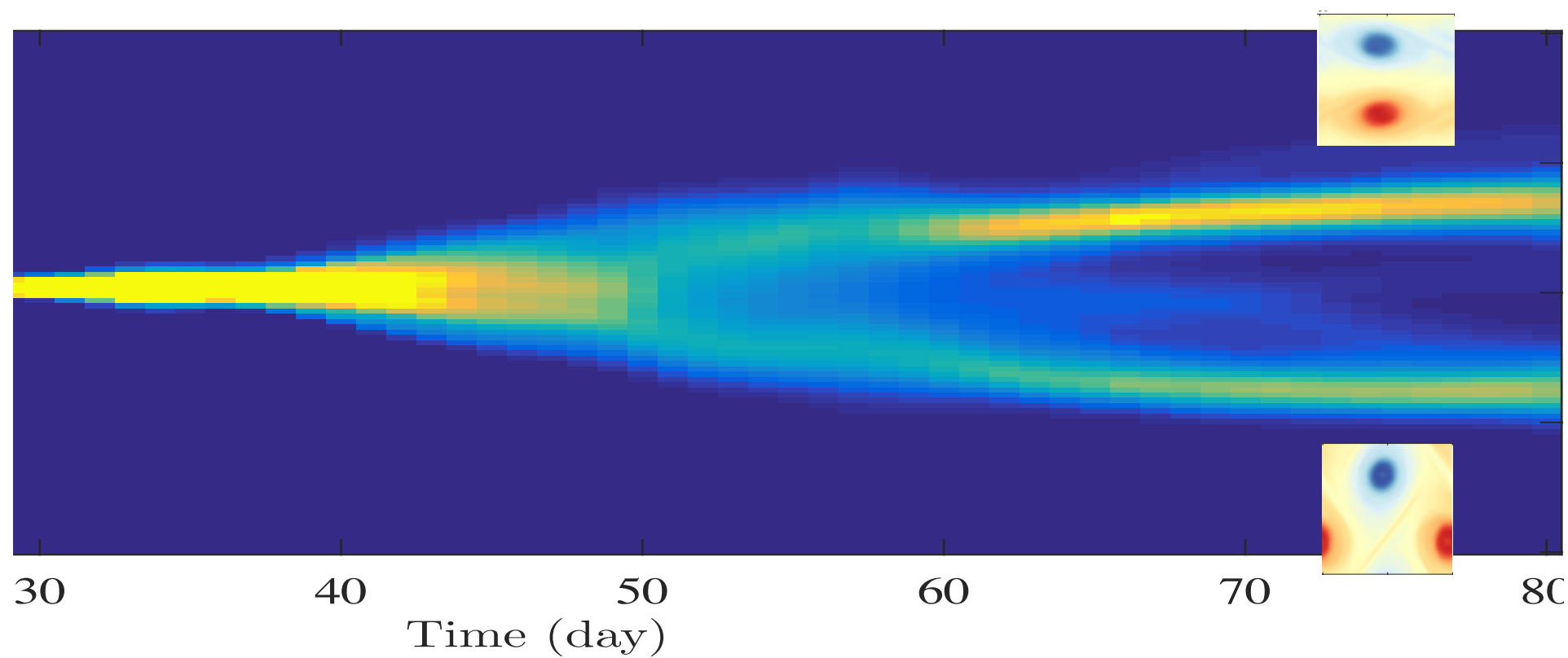
Under location
uncertainty



Random
initial
conditions



Under
location
uncertainty



Conclusion

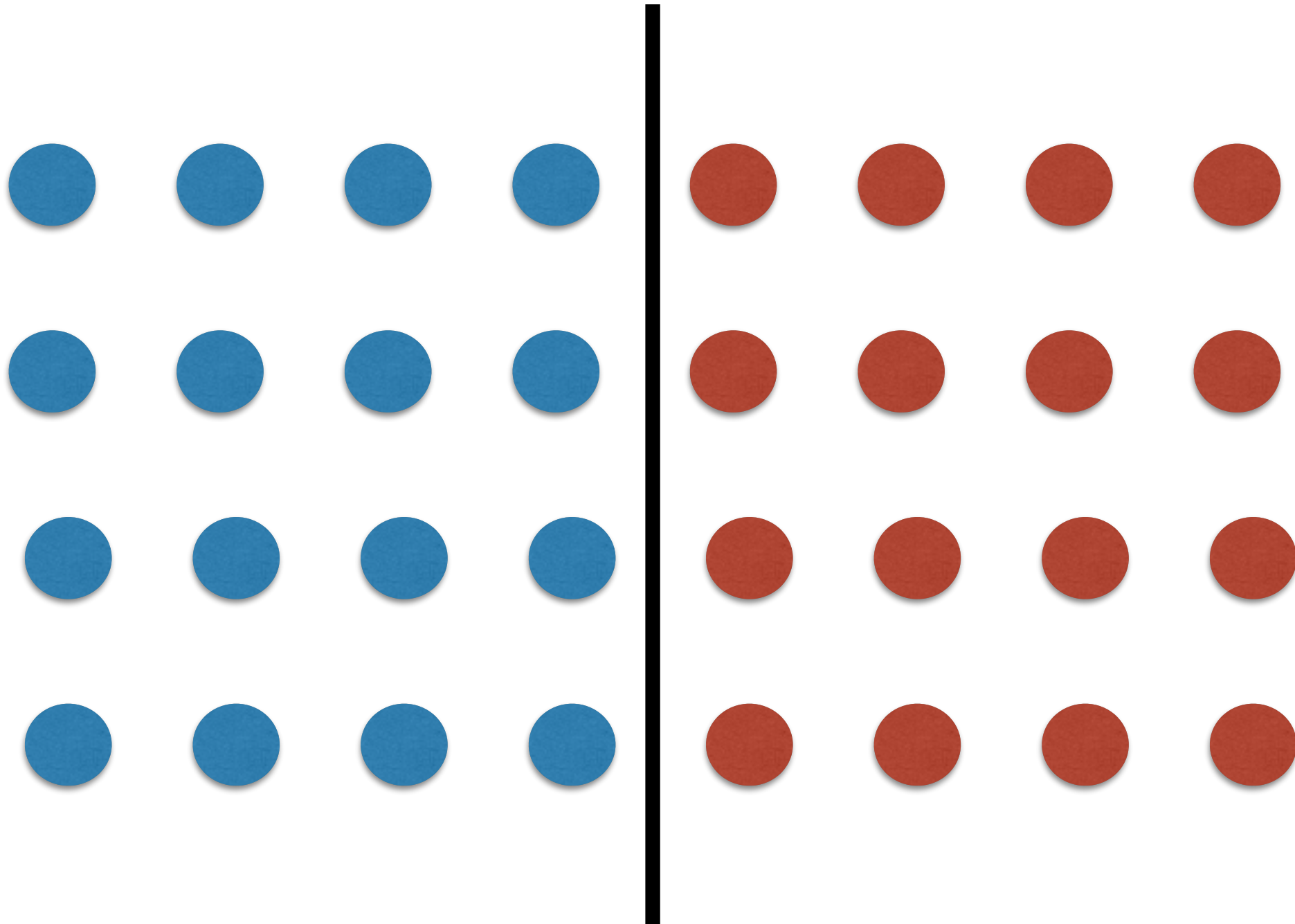
Conclusion

- Random transport applicable to any dynamics
- Link (inhomogeneous) diffusion, drift correction and multiplicative noise
- Predict likely scenarios with few realizations
- Other results:
 - Better small scales
 - Estimate positions and amplitudes of errors
 - Extreme events
 - Additional physical information

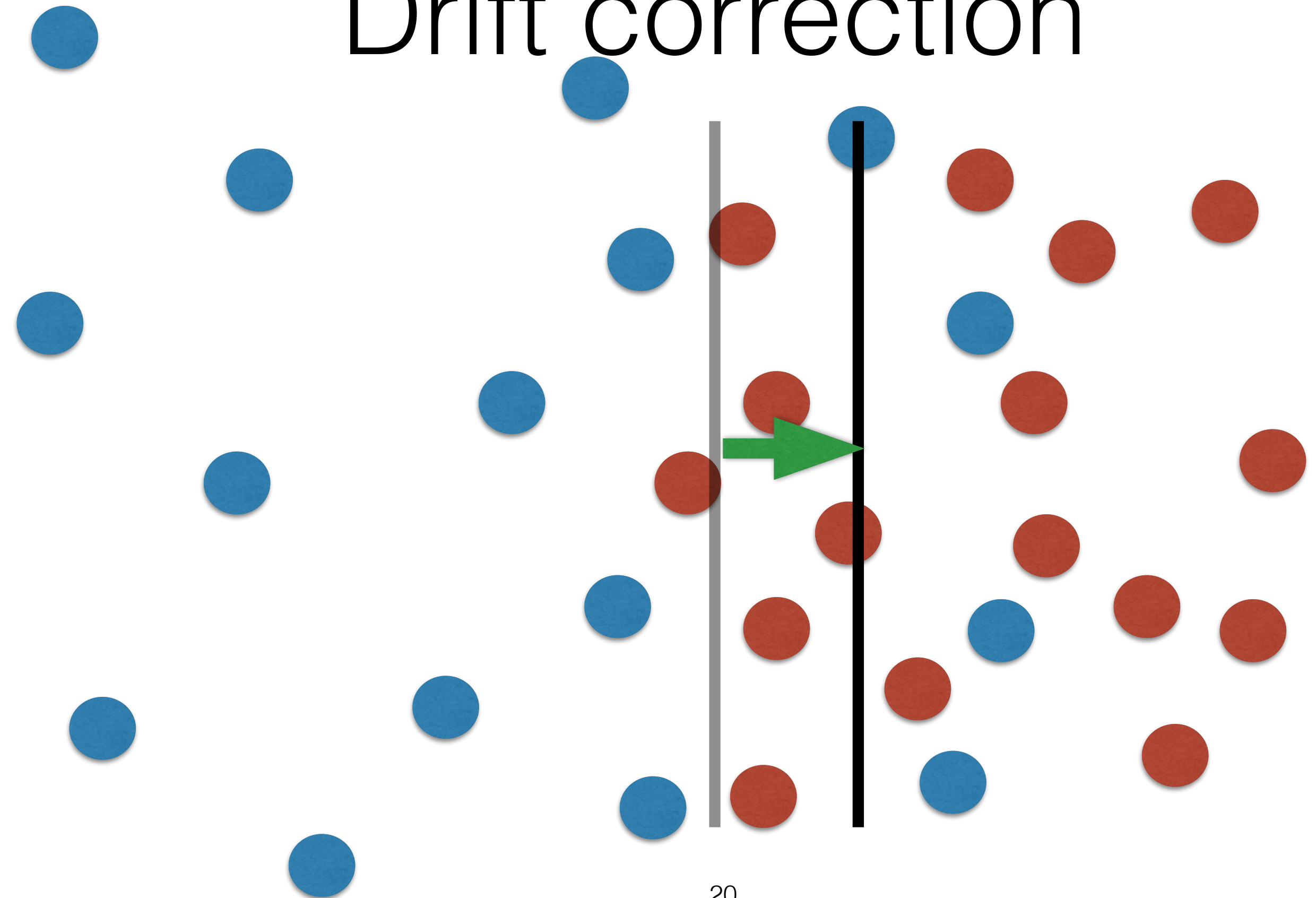
Thank you for your attention

Code SQG MU:
link from Fluminance website - V. Resseguier

Drift correction

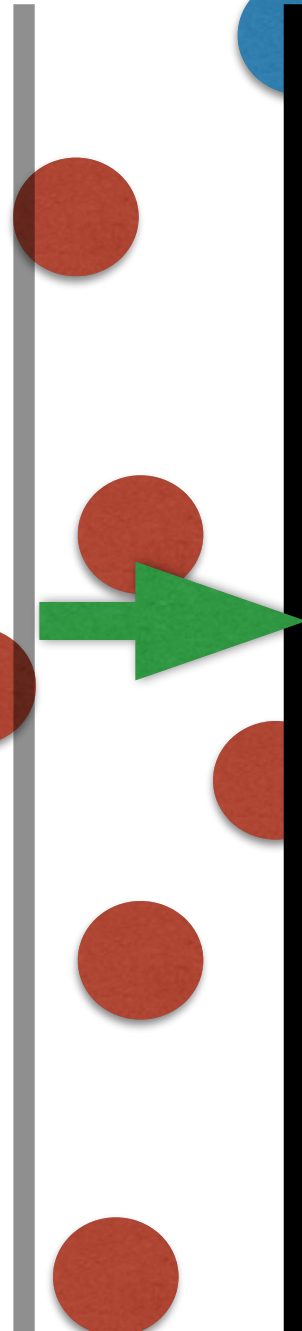


Drift correction



Drift correction

$$w^* = w - \frac{1}{2} (\nabla \cdot a)^T$$



SQG under Moderate Uncertainty

SQG MU

Code available online

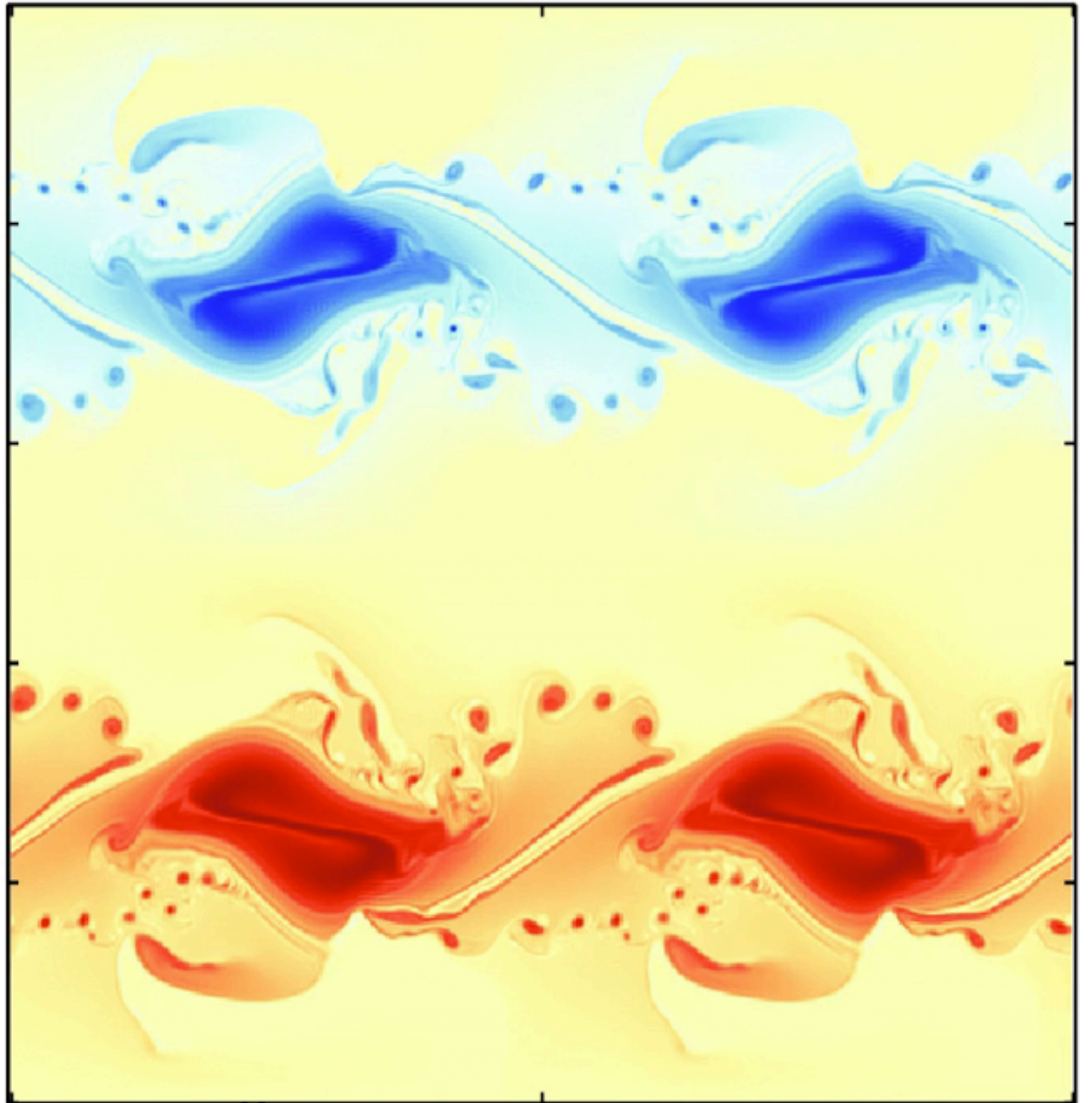
$t = 17$ days

Reference flow:

deterministic

SQG

512 x 512



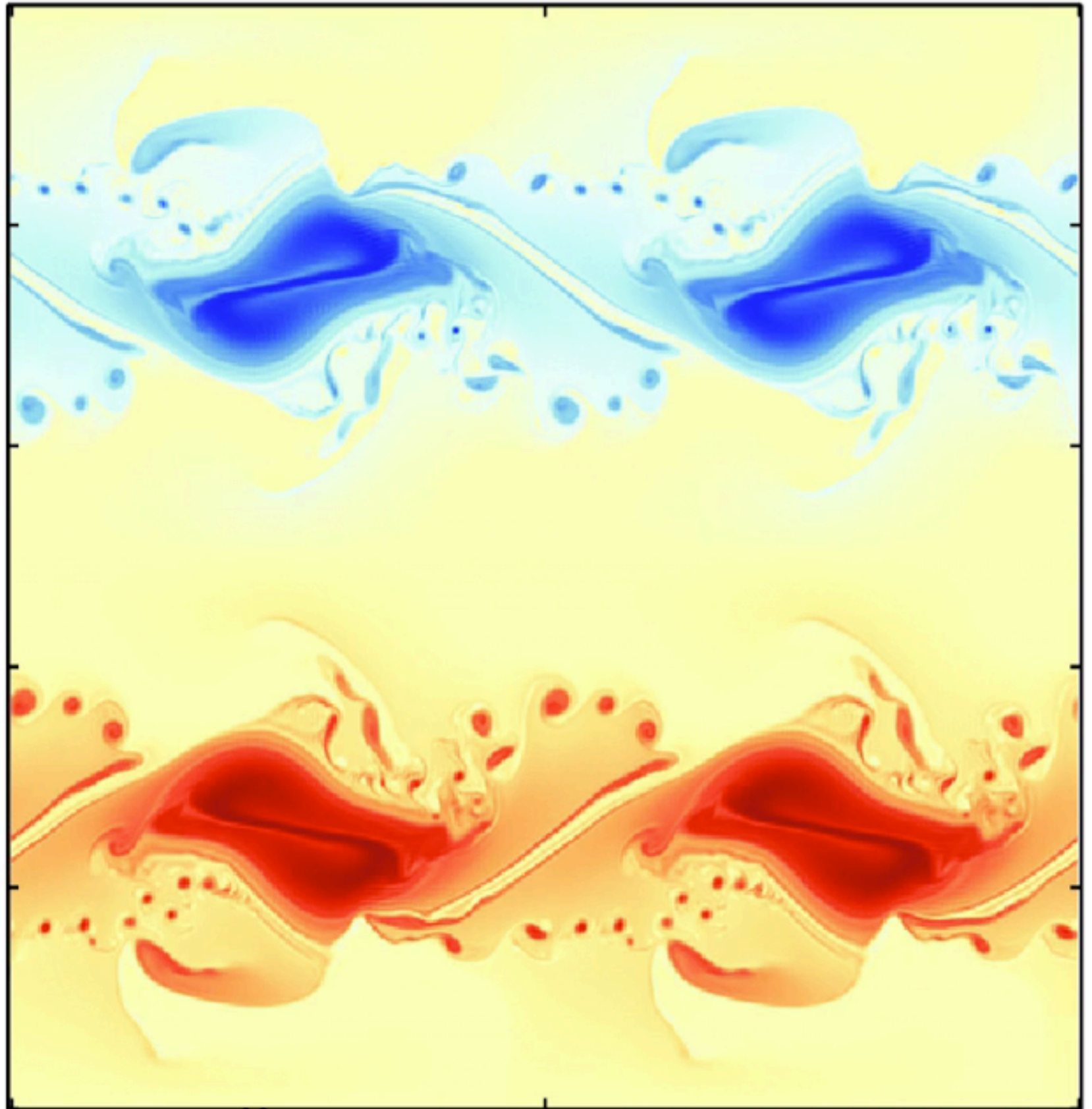
$t = 17$ days

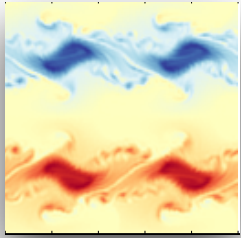
Reference flow:

deterministic

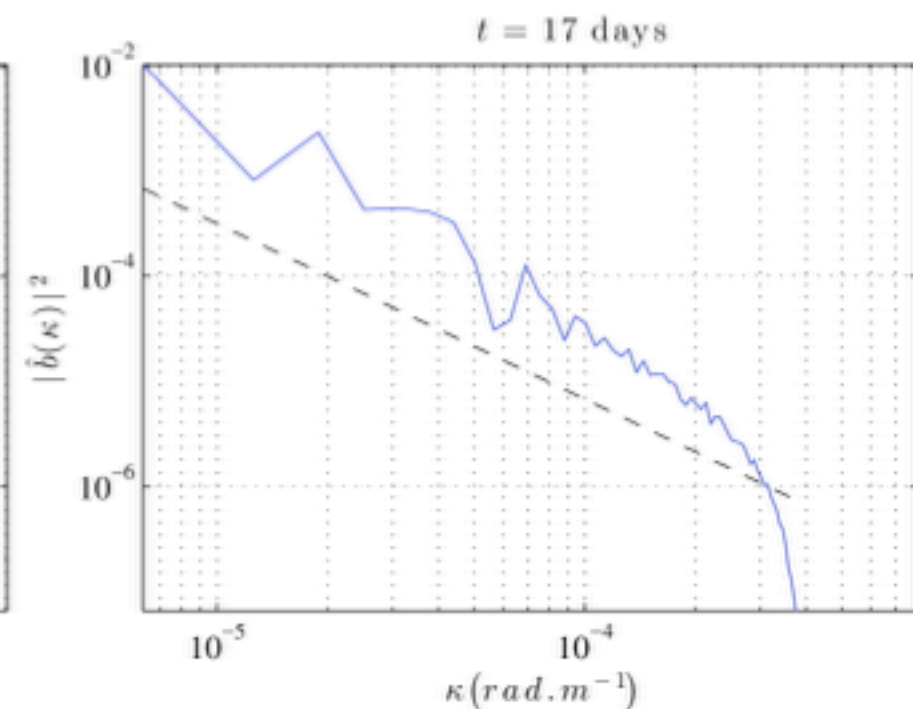
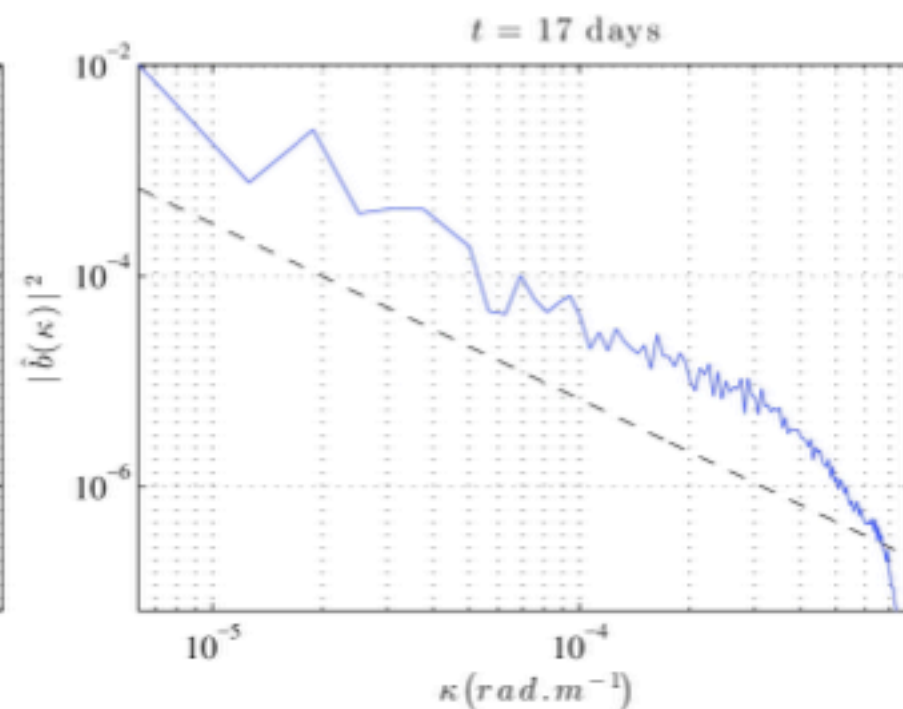
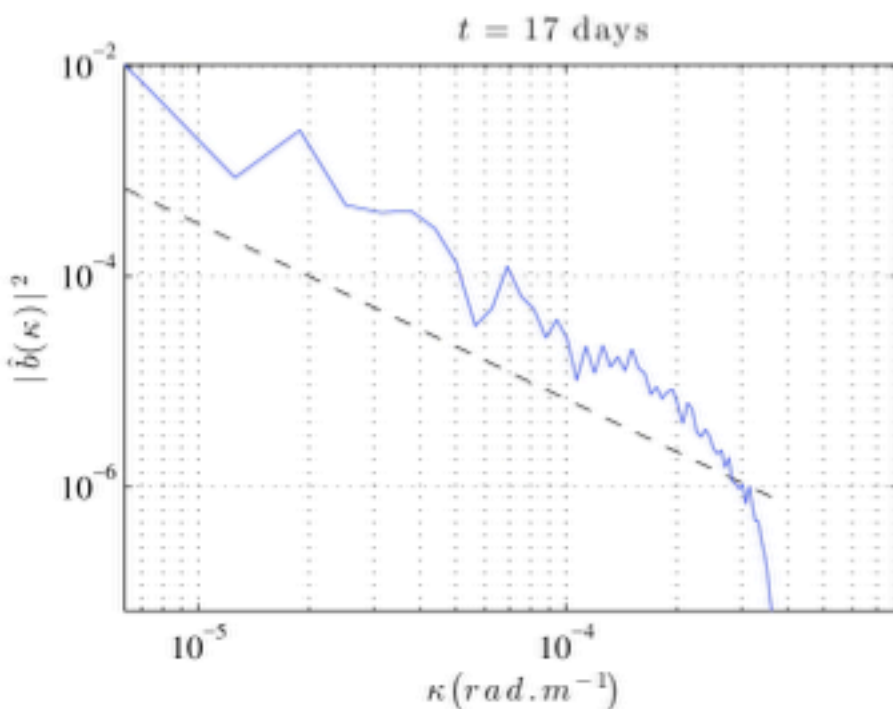
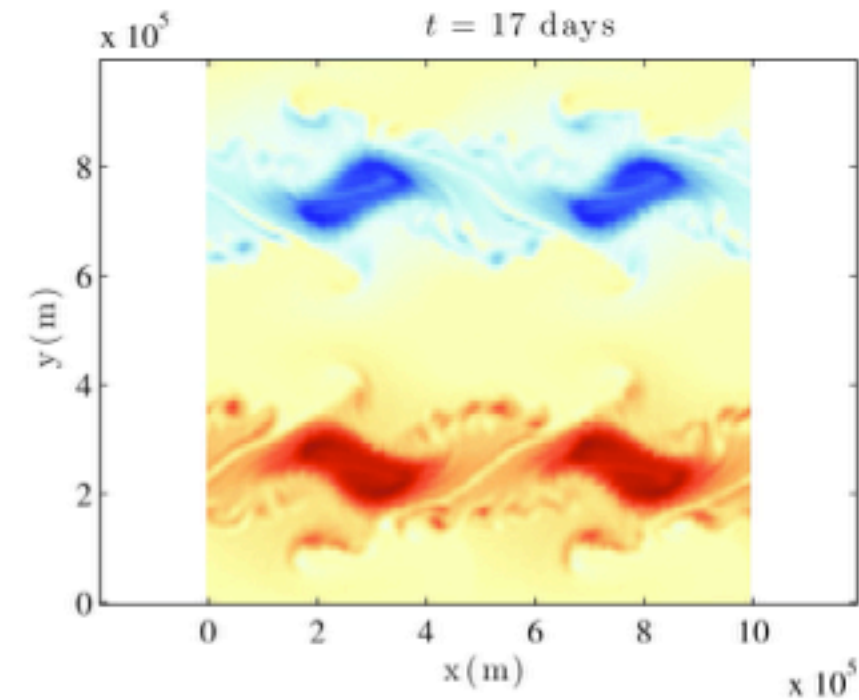
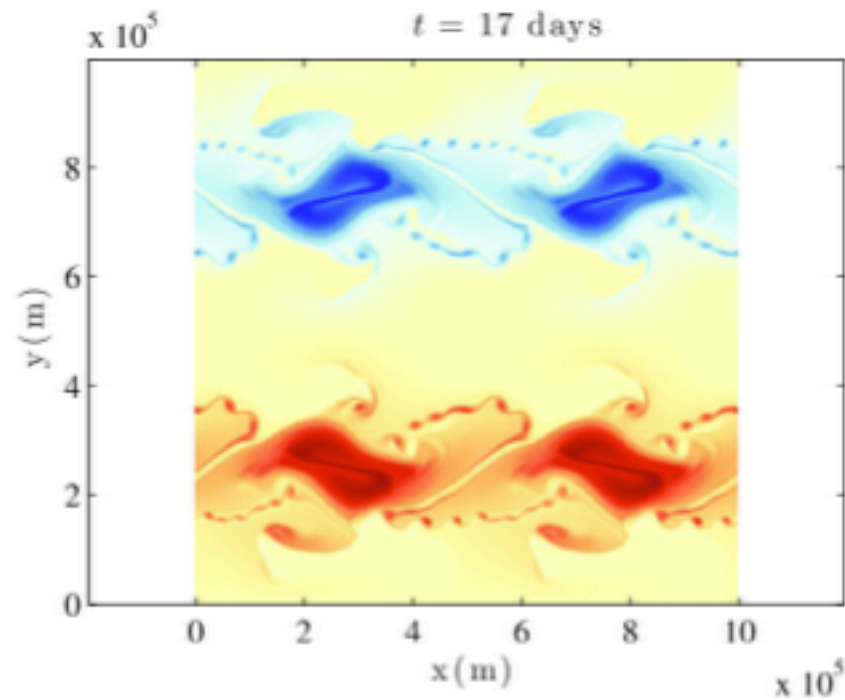
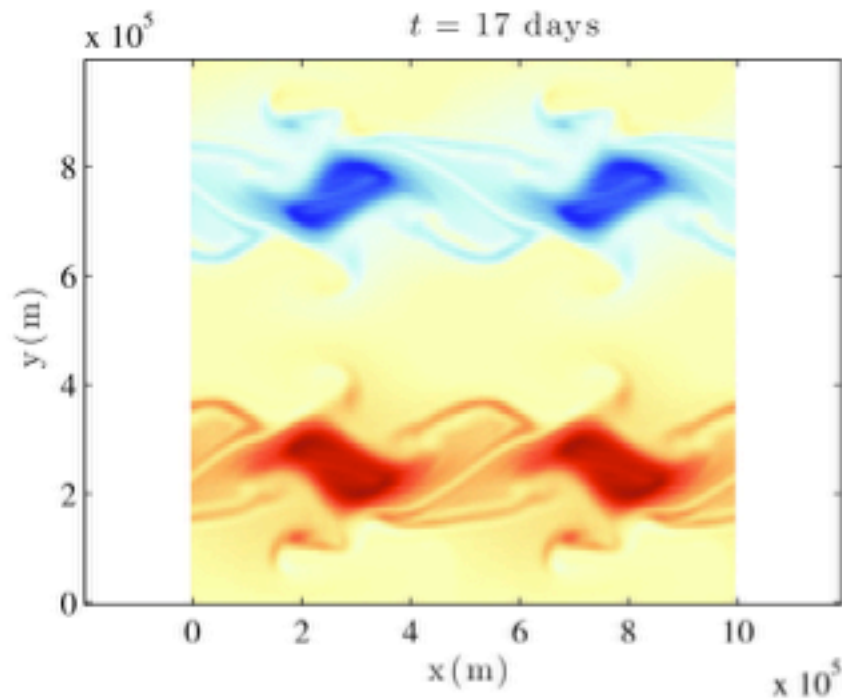
SQG

512 x 512





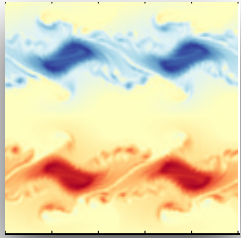
One realization



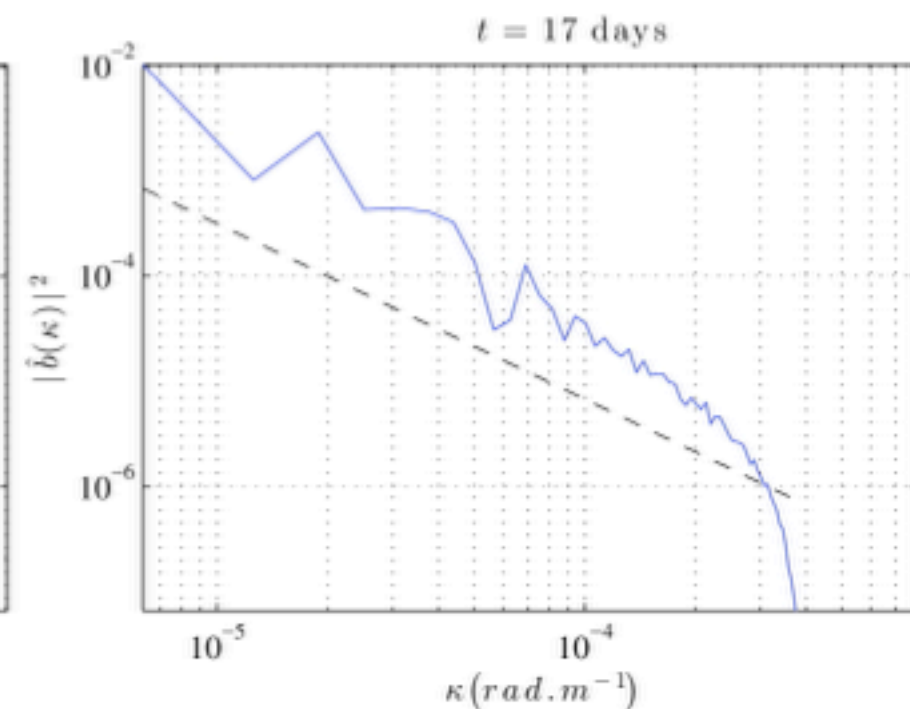
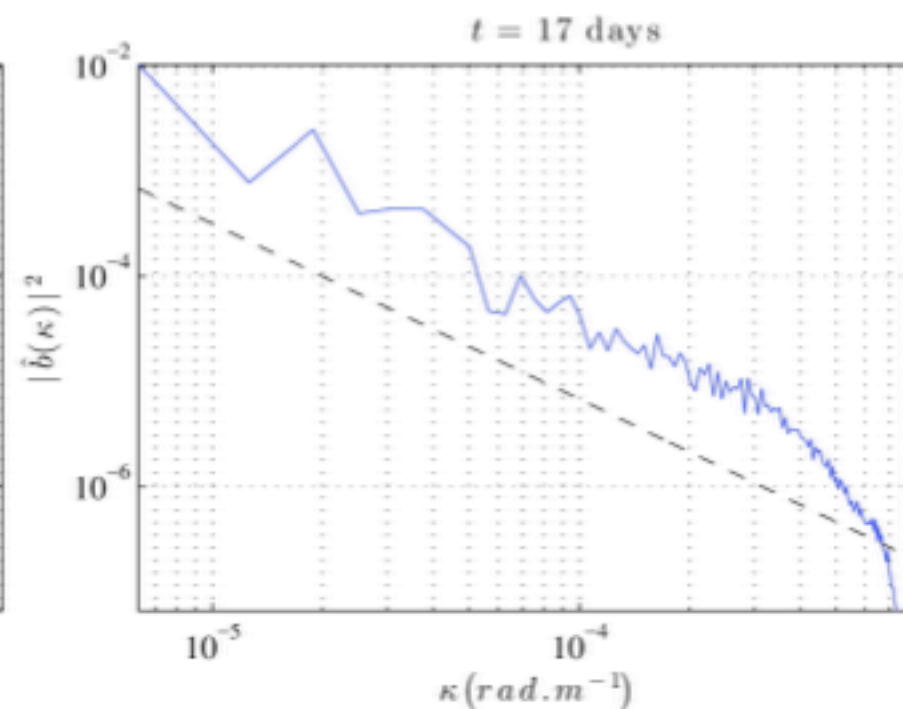
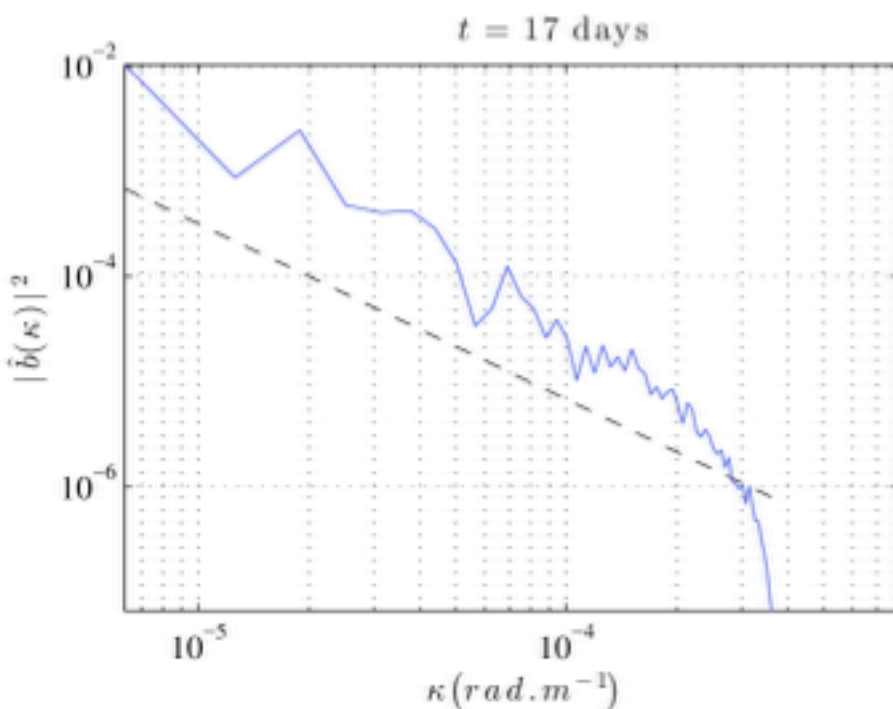
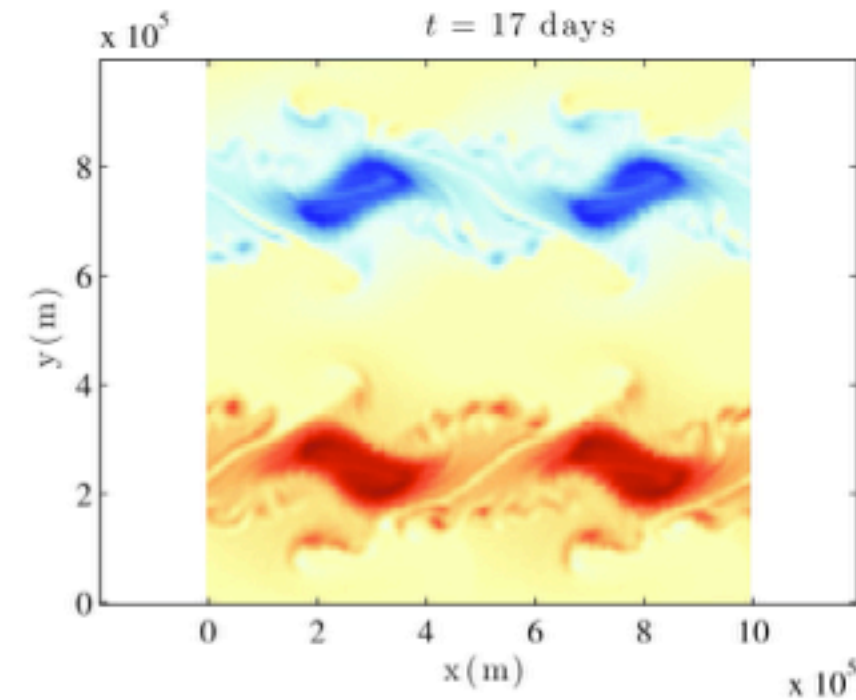
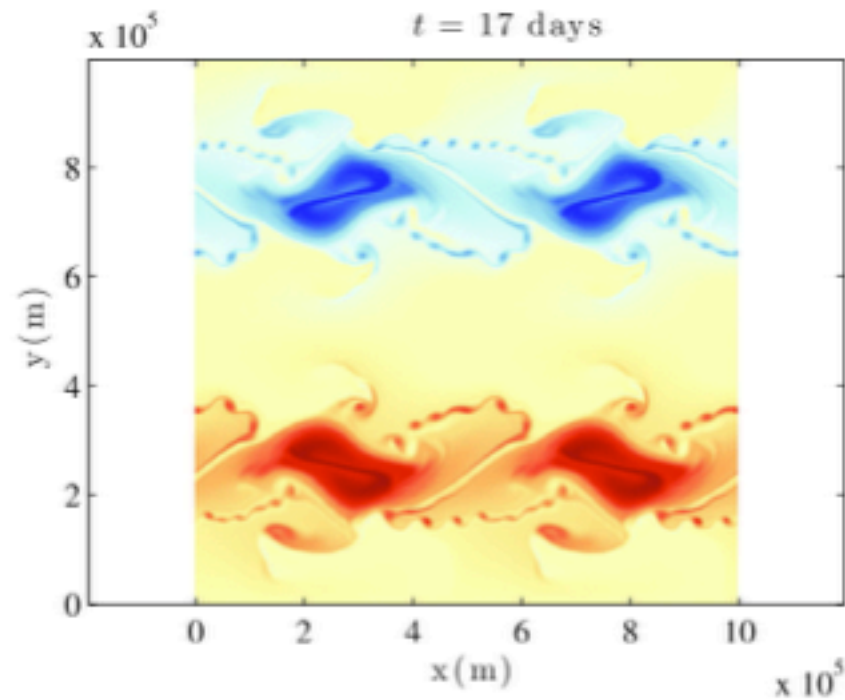
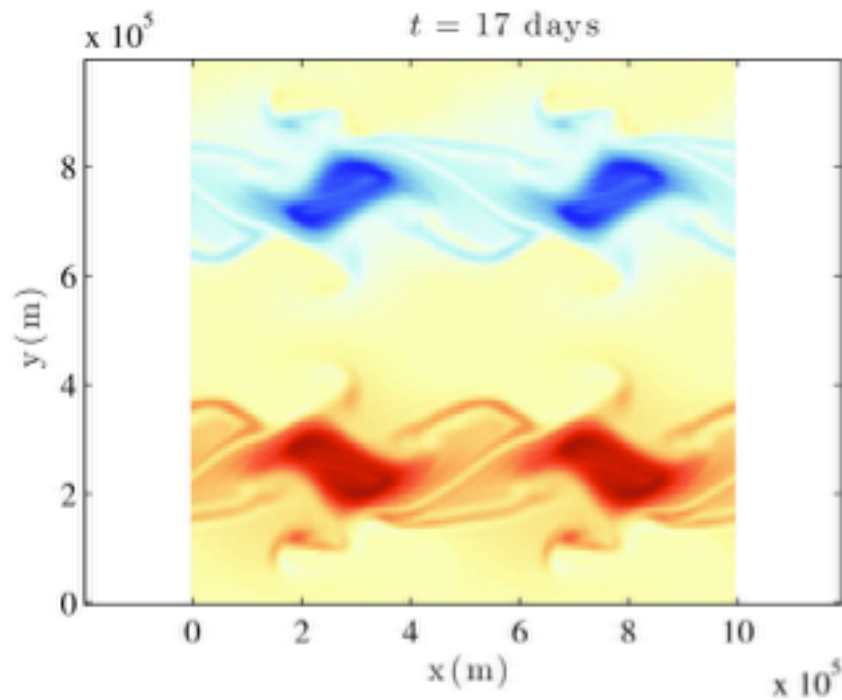
Deterministic 128x128

Deterministic 512x512

Stochastic 128x128



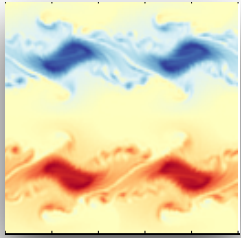
One realization



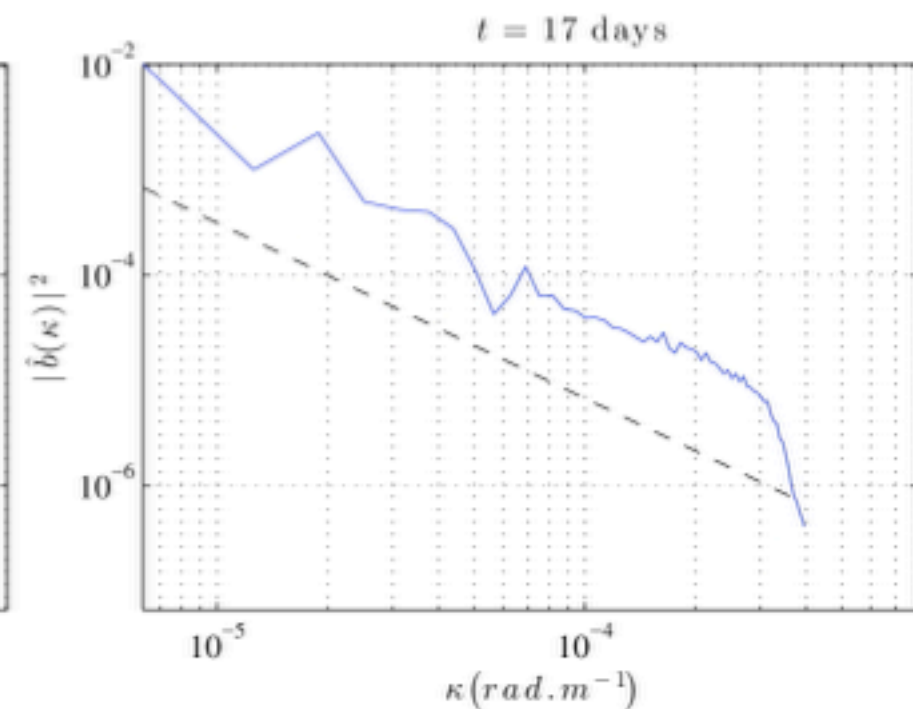
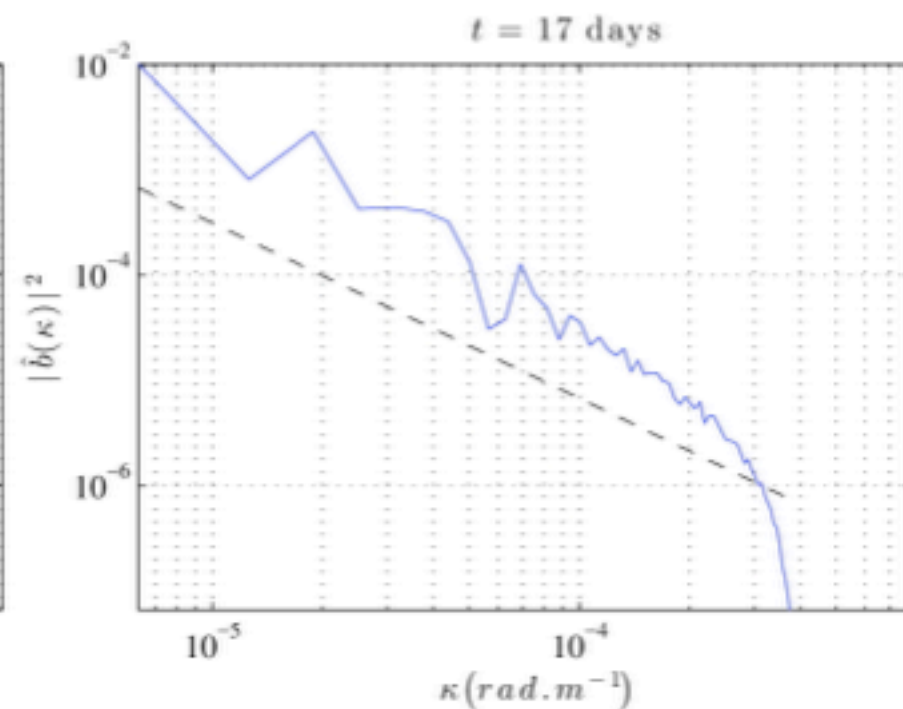
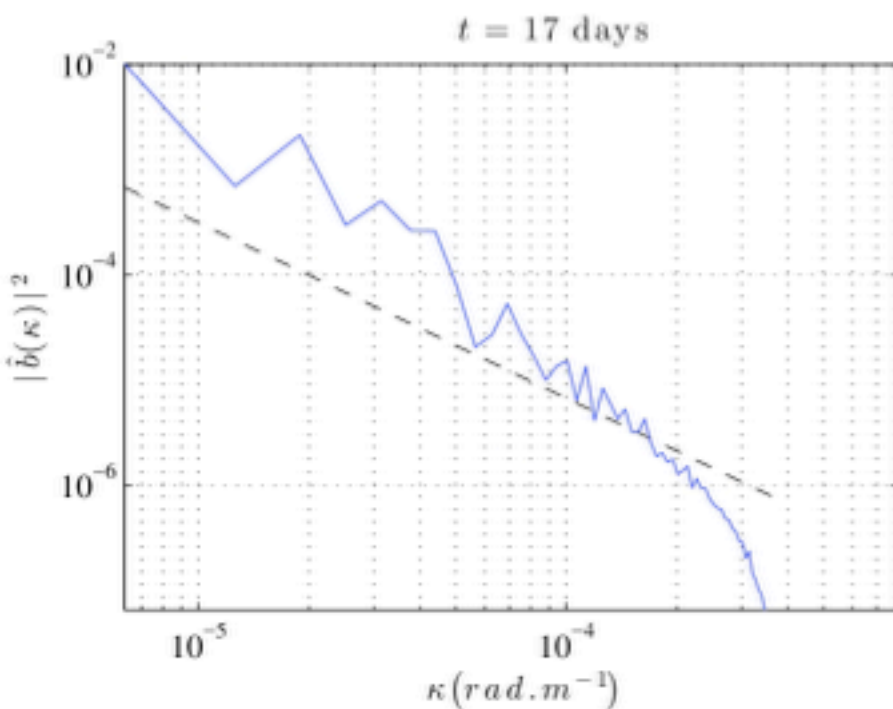
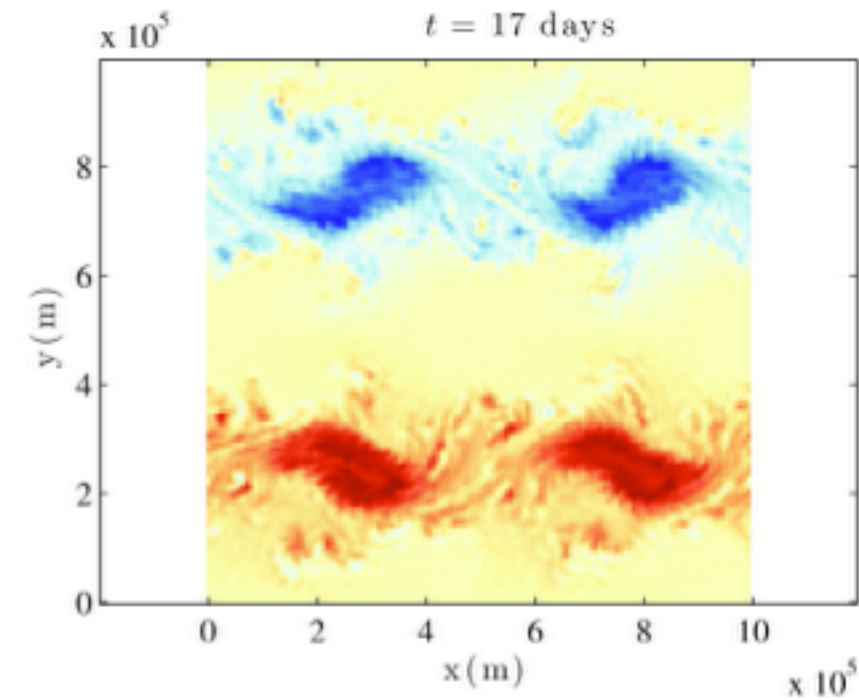
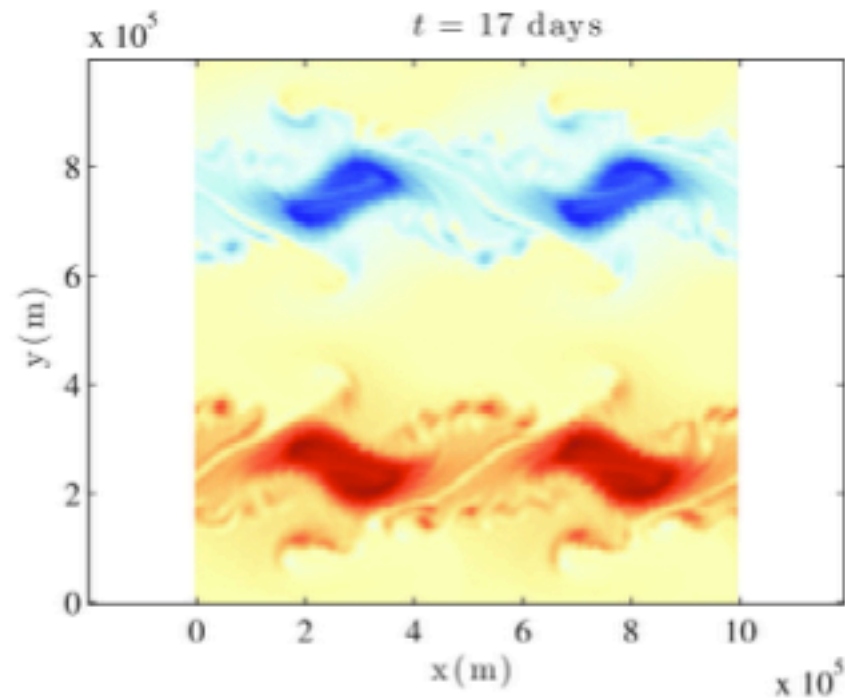
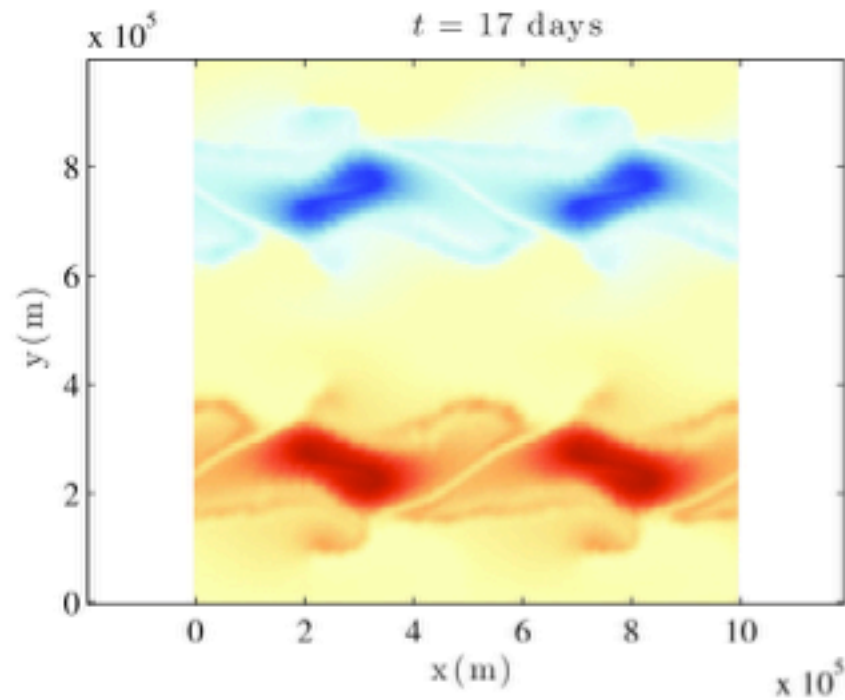
Deterministic 128x128

Deterministic 512x512

Stochastic 128x128



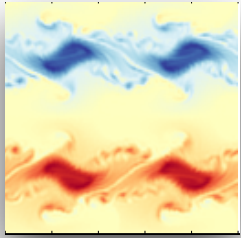
One realization



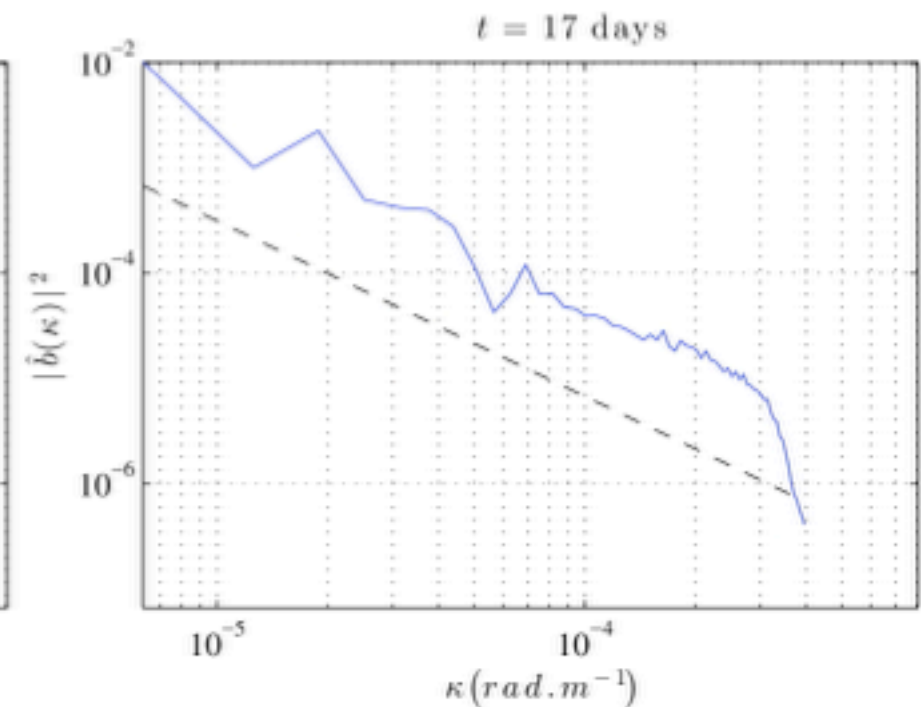
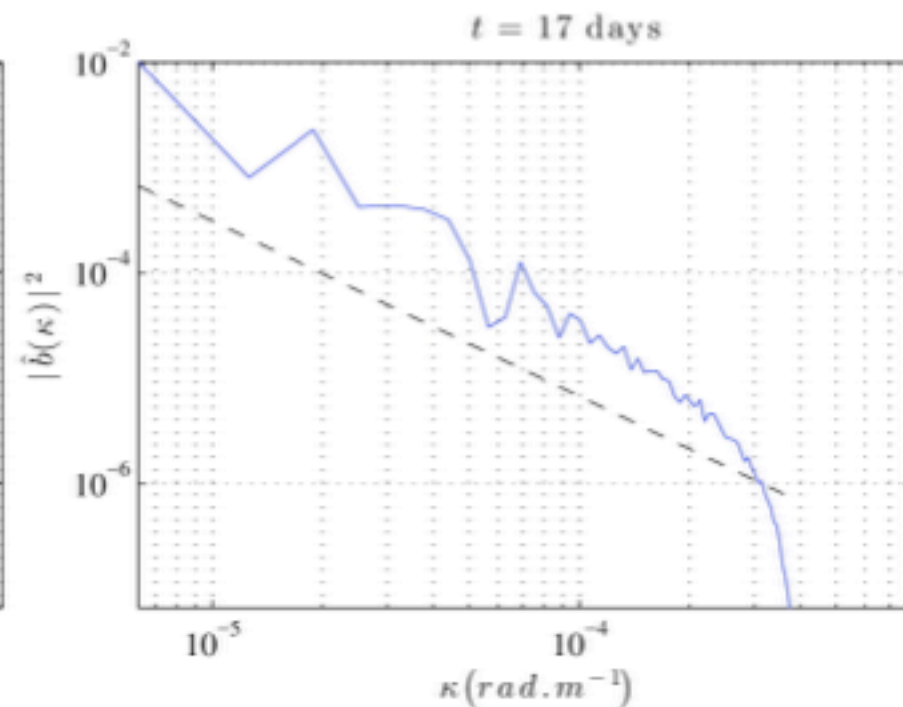
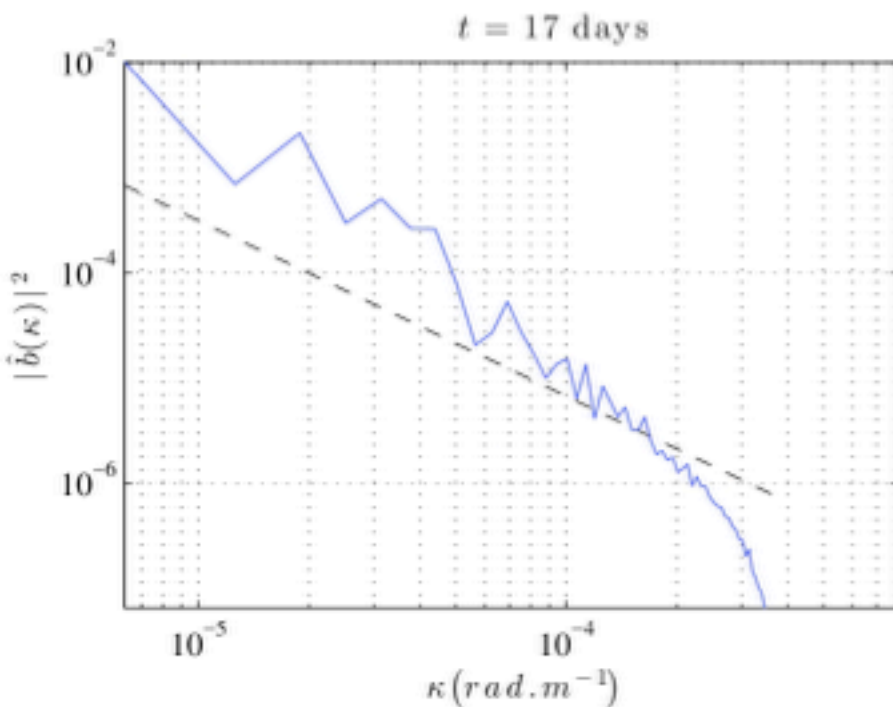
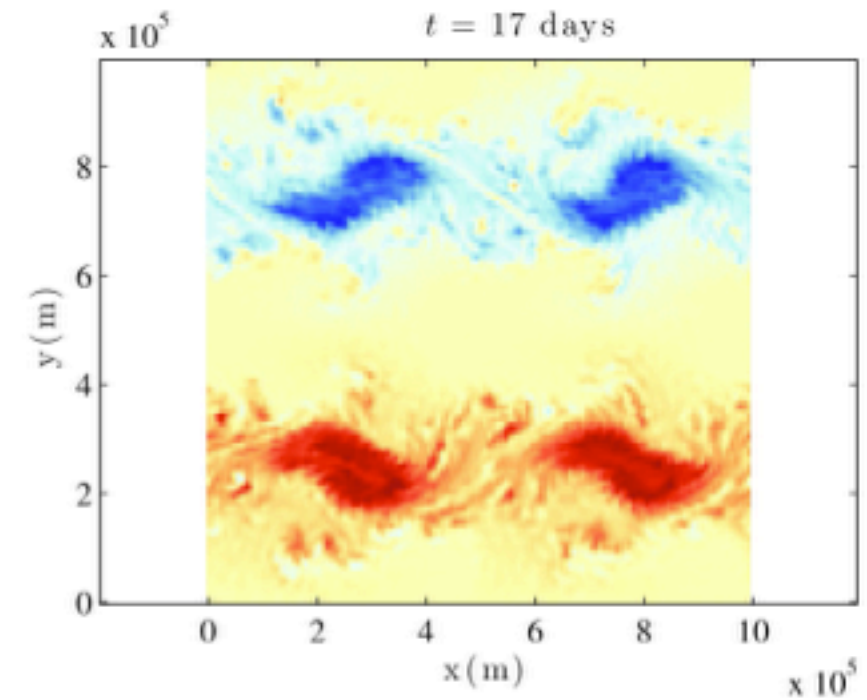
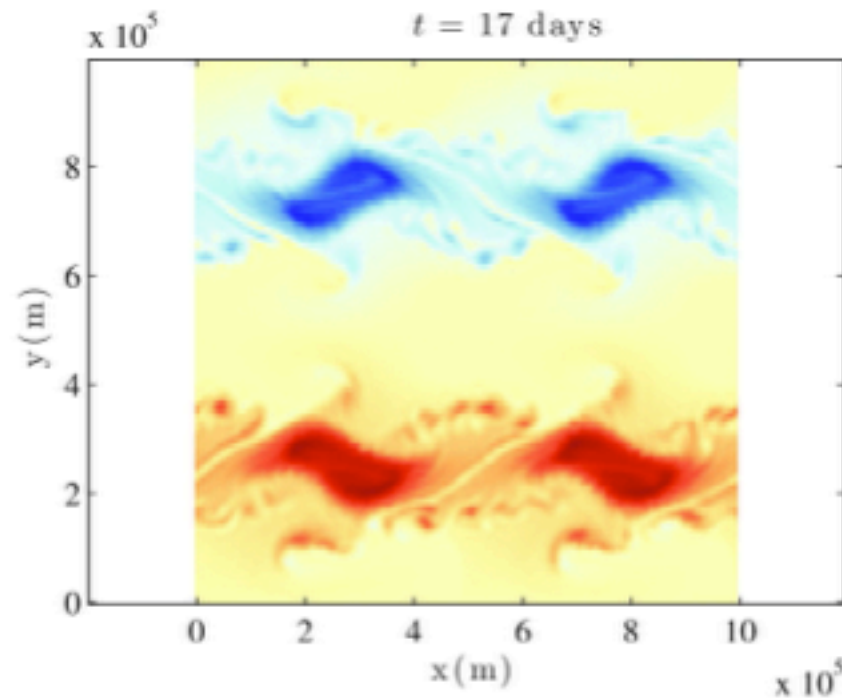
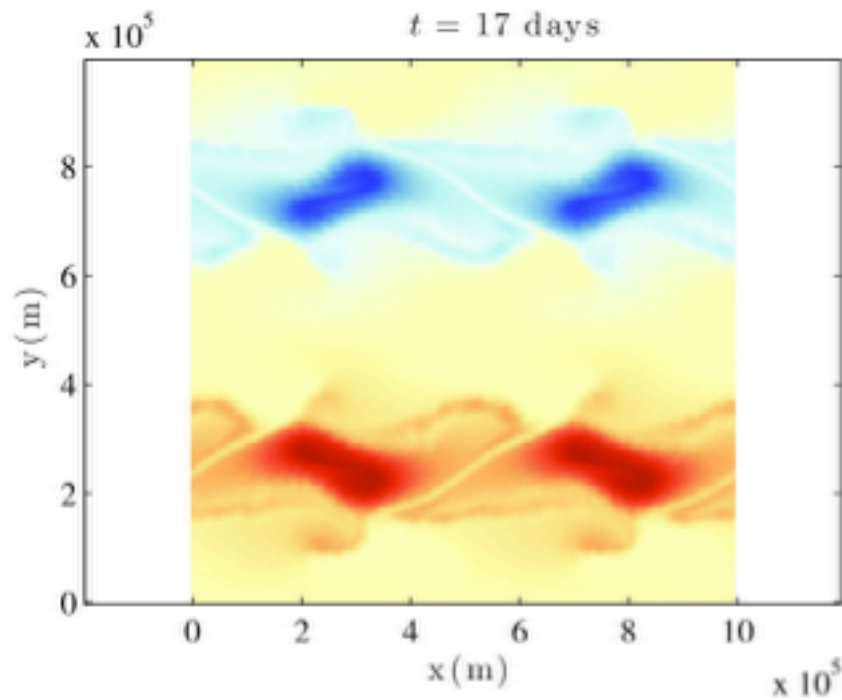
Lower noise

Our model

Larger noise



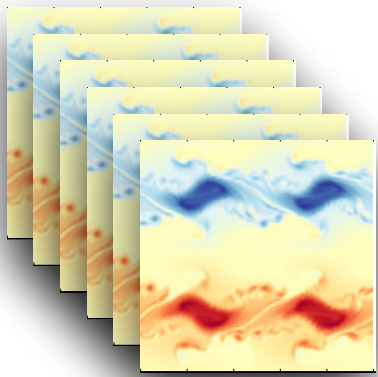
One realization



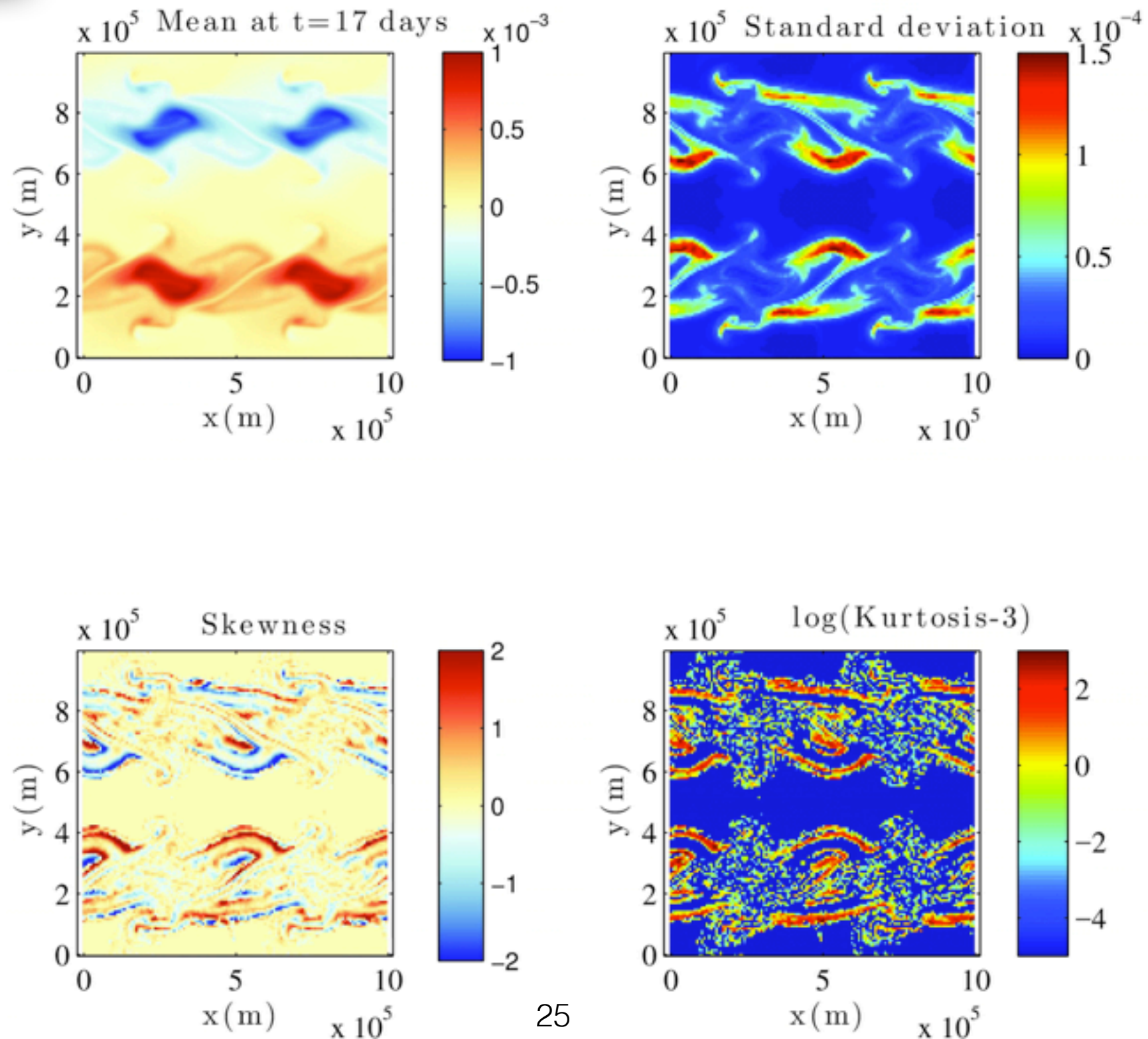
Lower noise

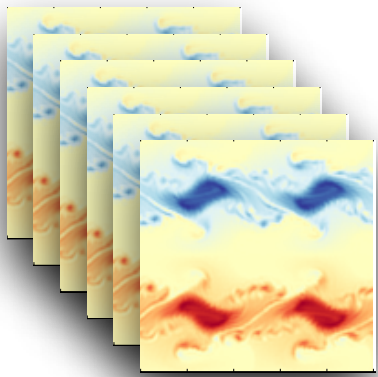
Our model

Larger noise

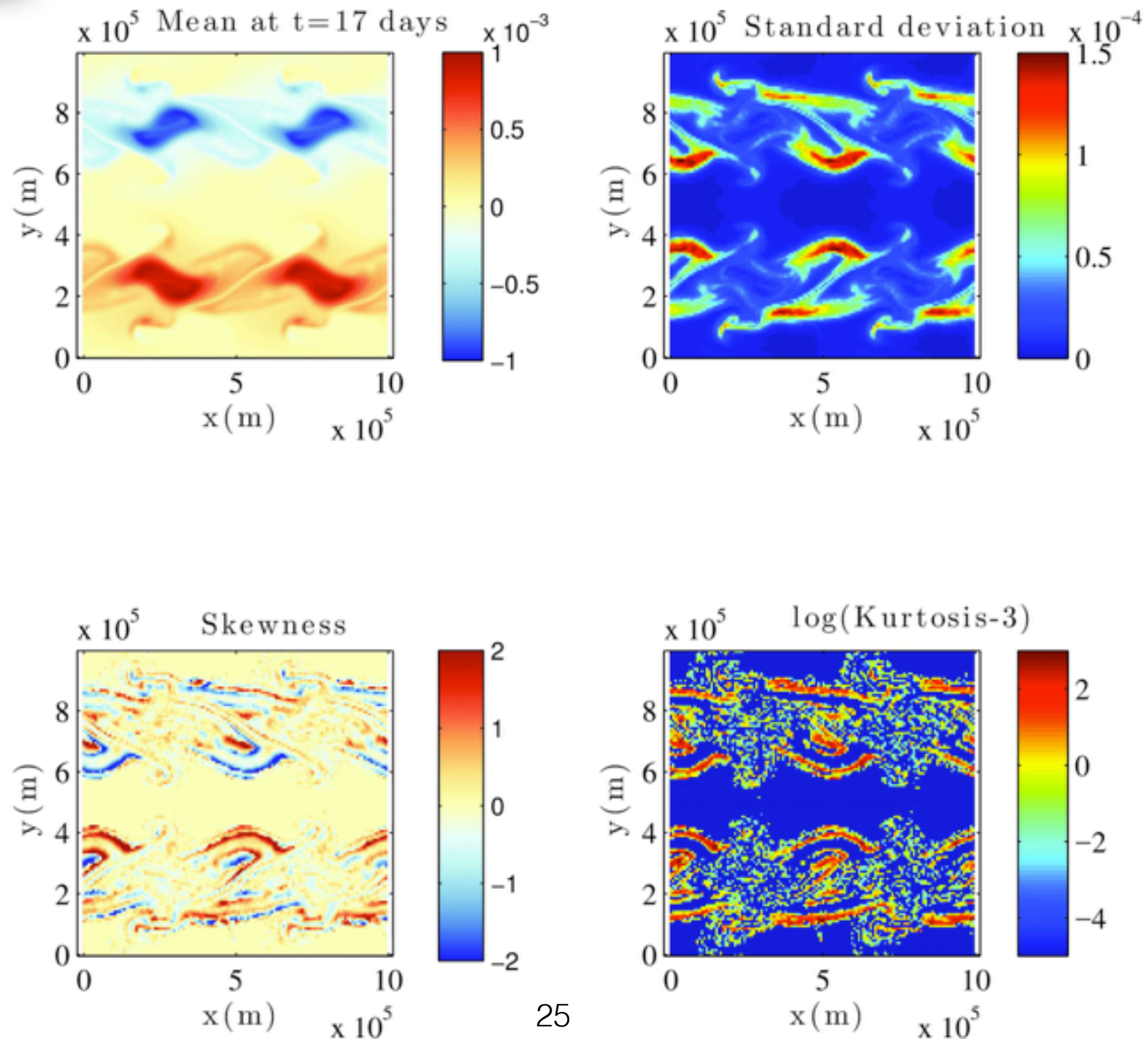


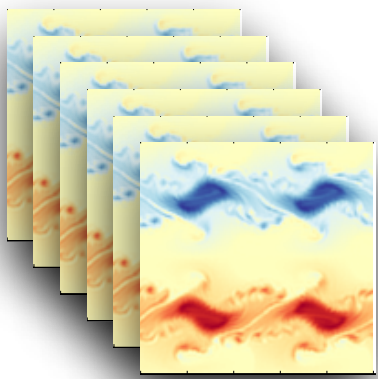
Ensemble



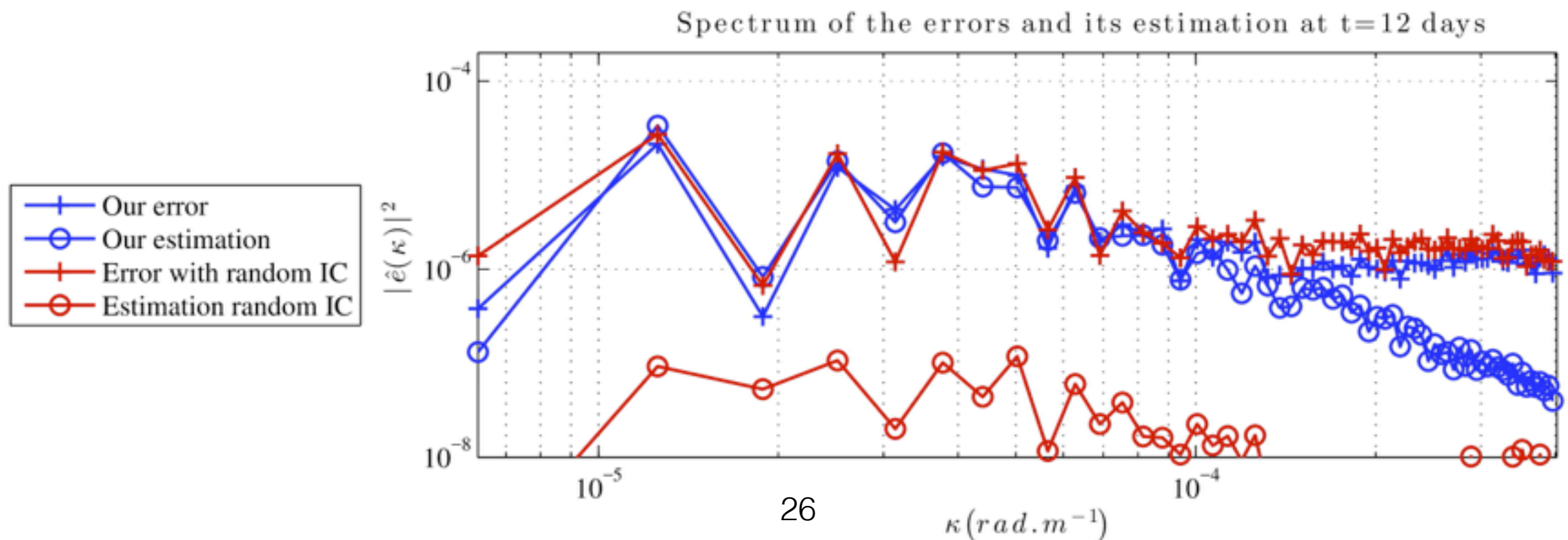
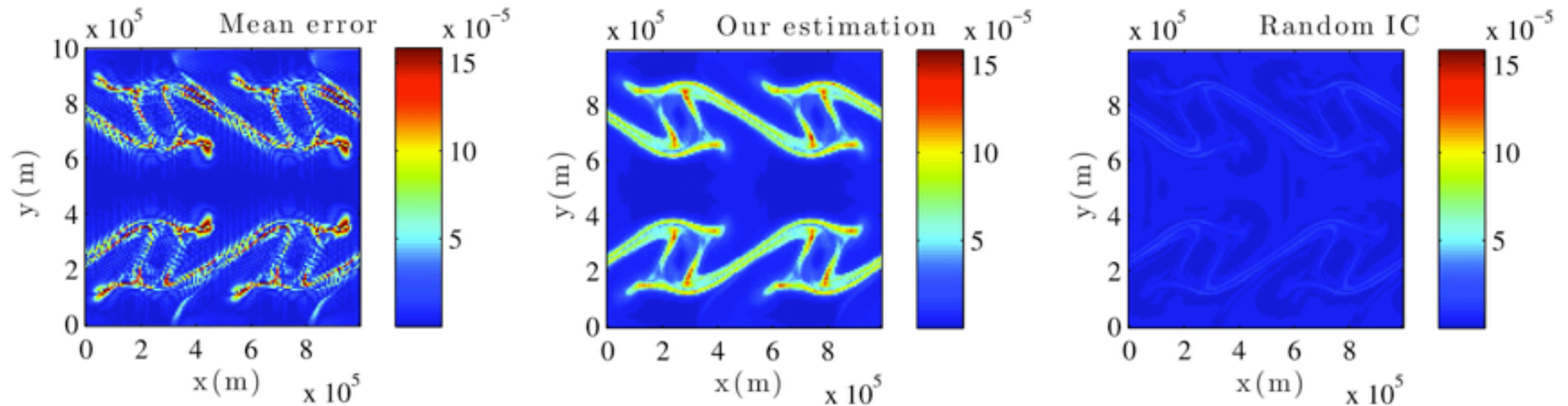


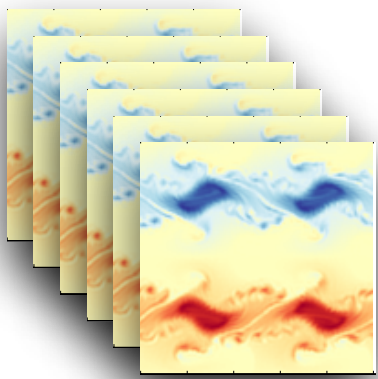
Ensemble



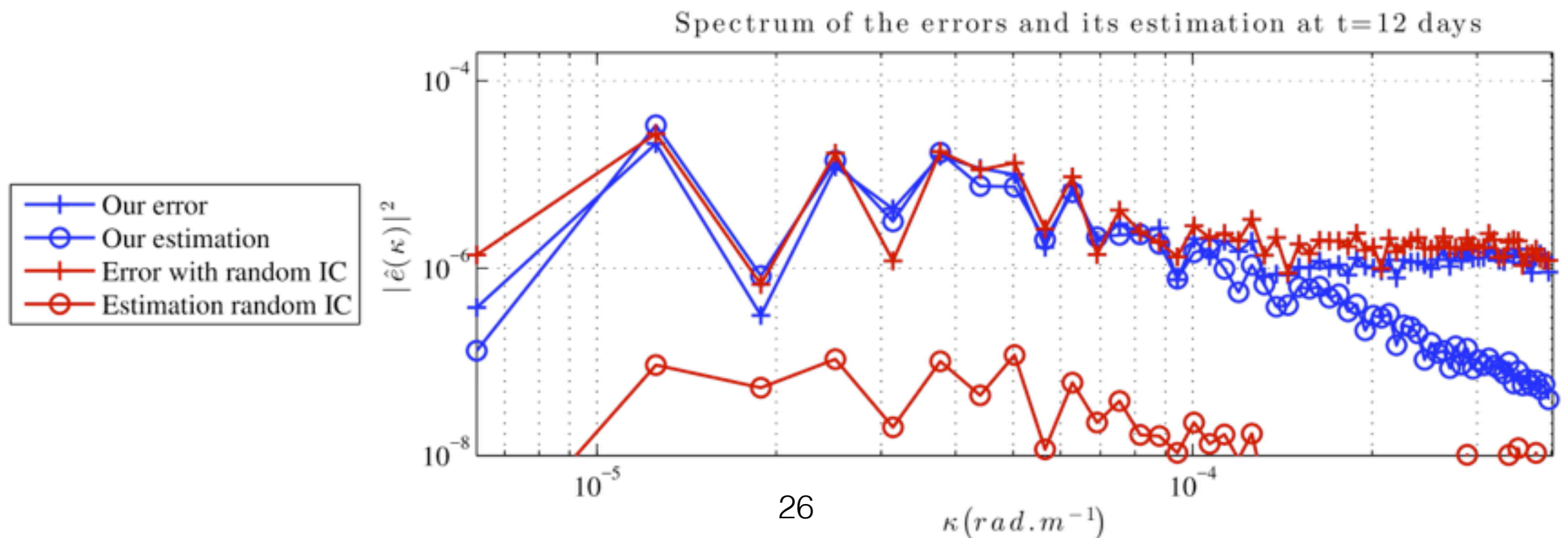
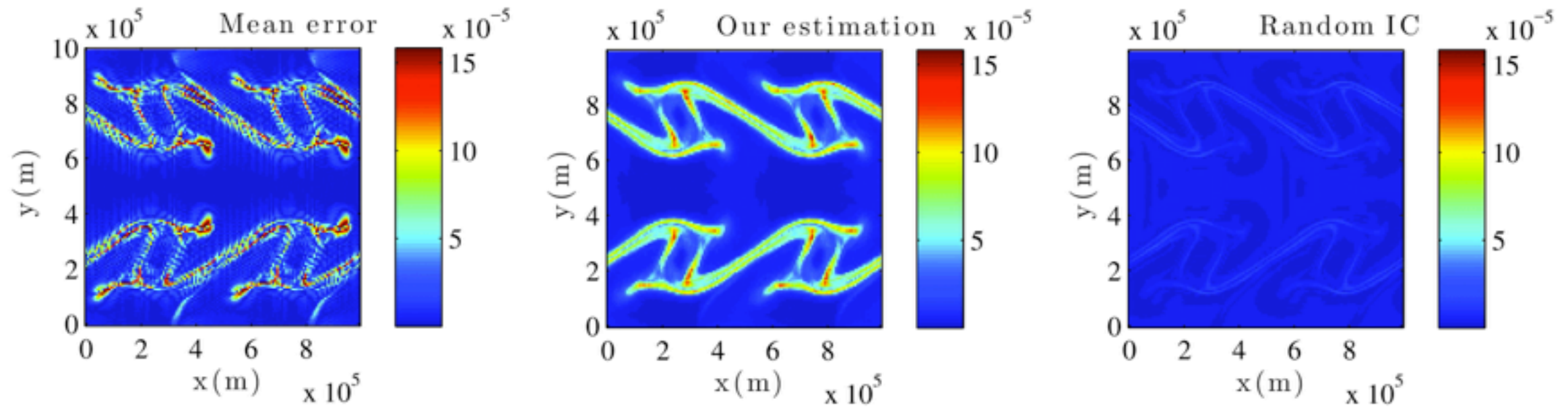


Ensemble





Ensemble



Likely SQG scenarios

tracked by SQG MU

SQG under Strong Uncertainty

SQG SU

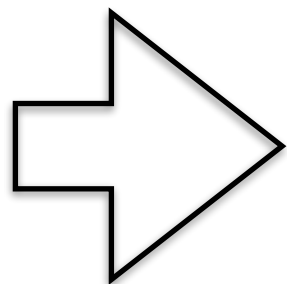
Mesoscale divergence

Geostrophic balance

$$\mathbf{f} \times \mathbf{u} = -\frac{1}{\rho_b} \nabla p'$$

Horizontal
Diffusion

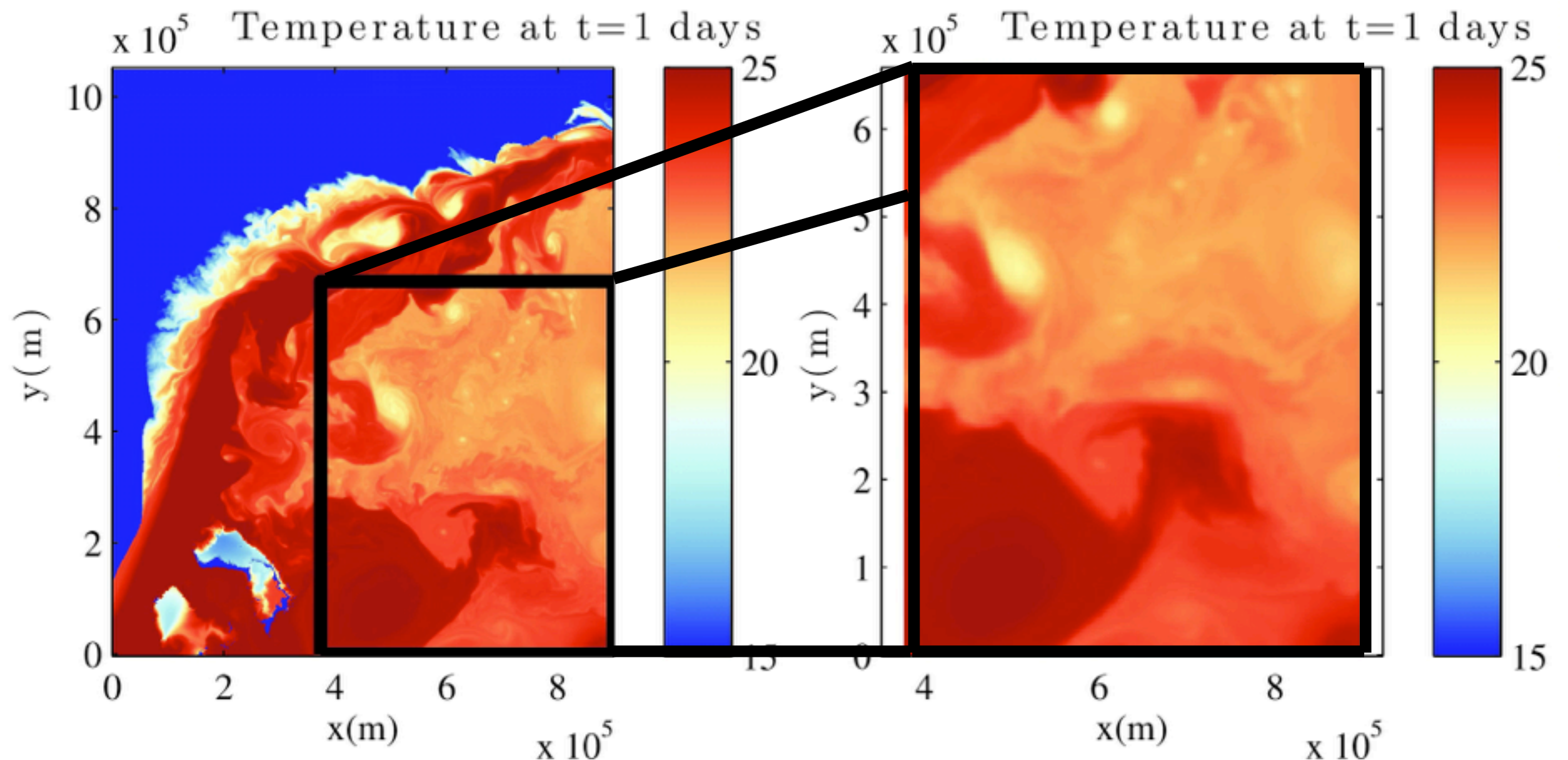
$$+ \frac{a}{2} \Delta \mathbf{u}$$



$$\nabla \cdot \mathbf{u} \propto \Delta \nabla^\perp \cdot \mathbf{u}$$

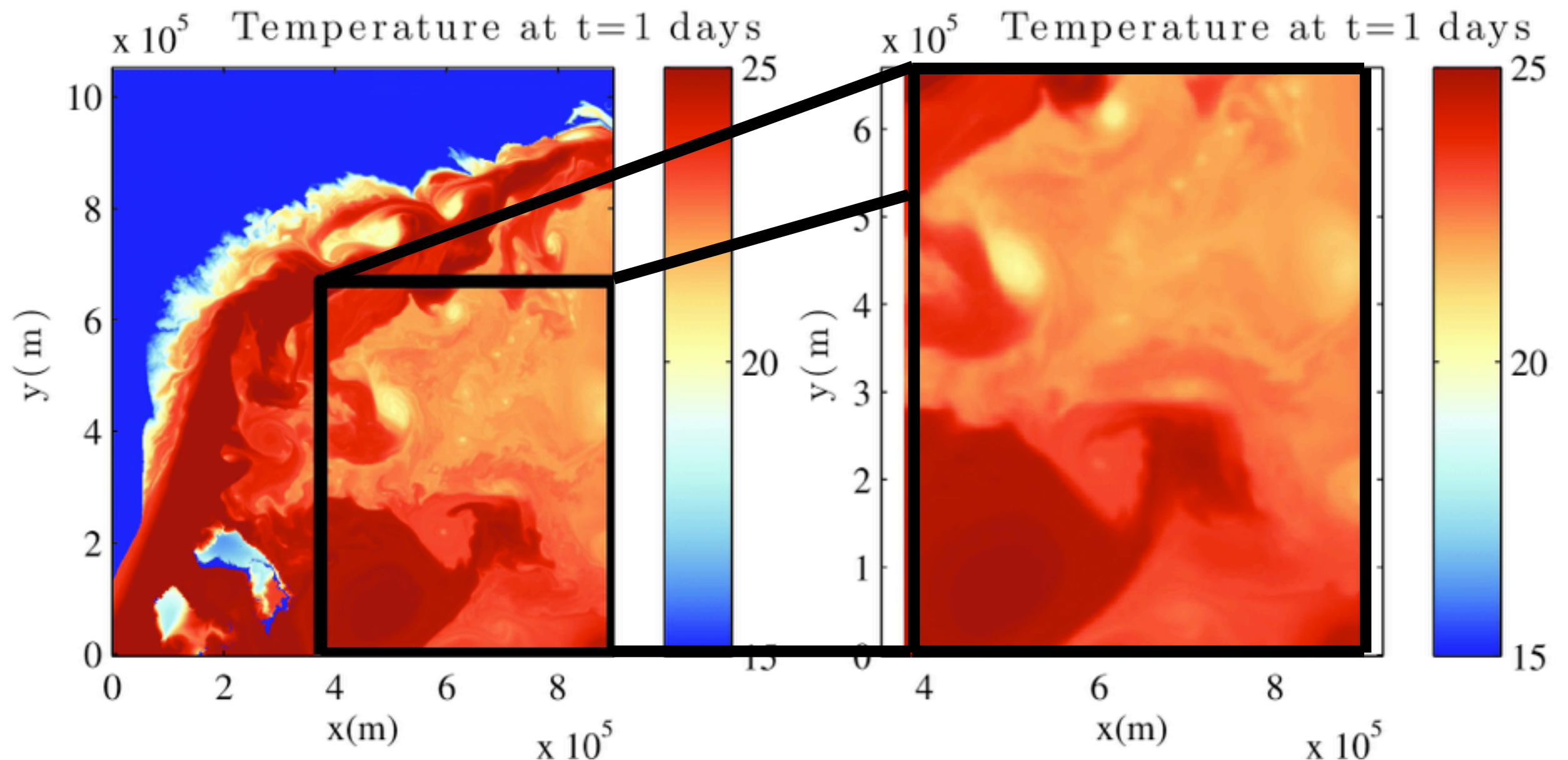
Filtering of model outputs:

Gula, Jonathan, M. Jeroen Molemaker, and James C. McWilliams
"Gulf Stream dynamics along the southeastern US seaboard."
Journal of Physical Oceanography 45.3 (2015): 690-715.

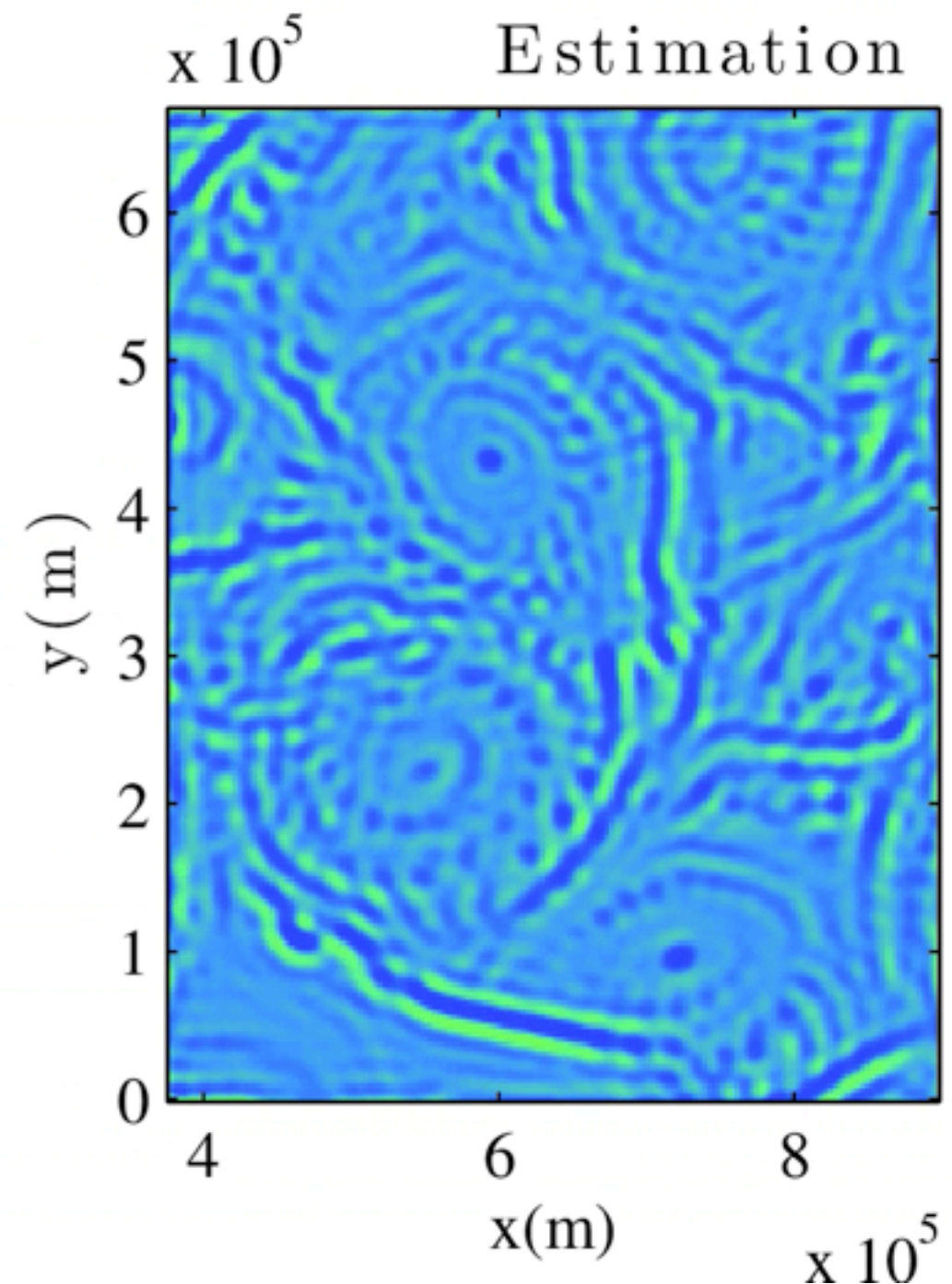
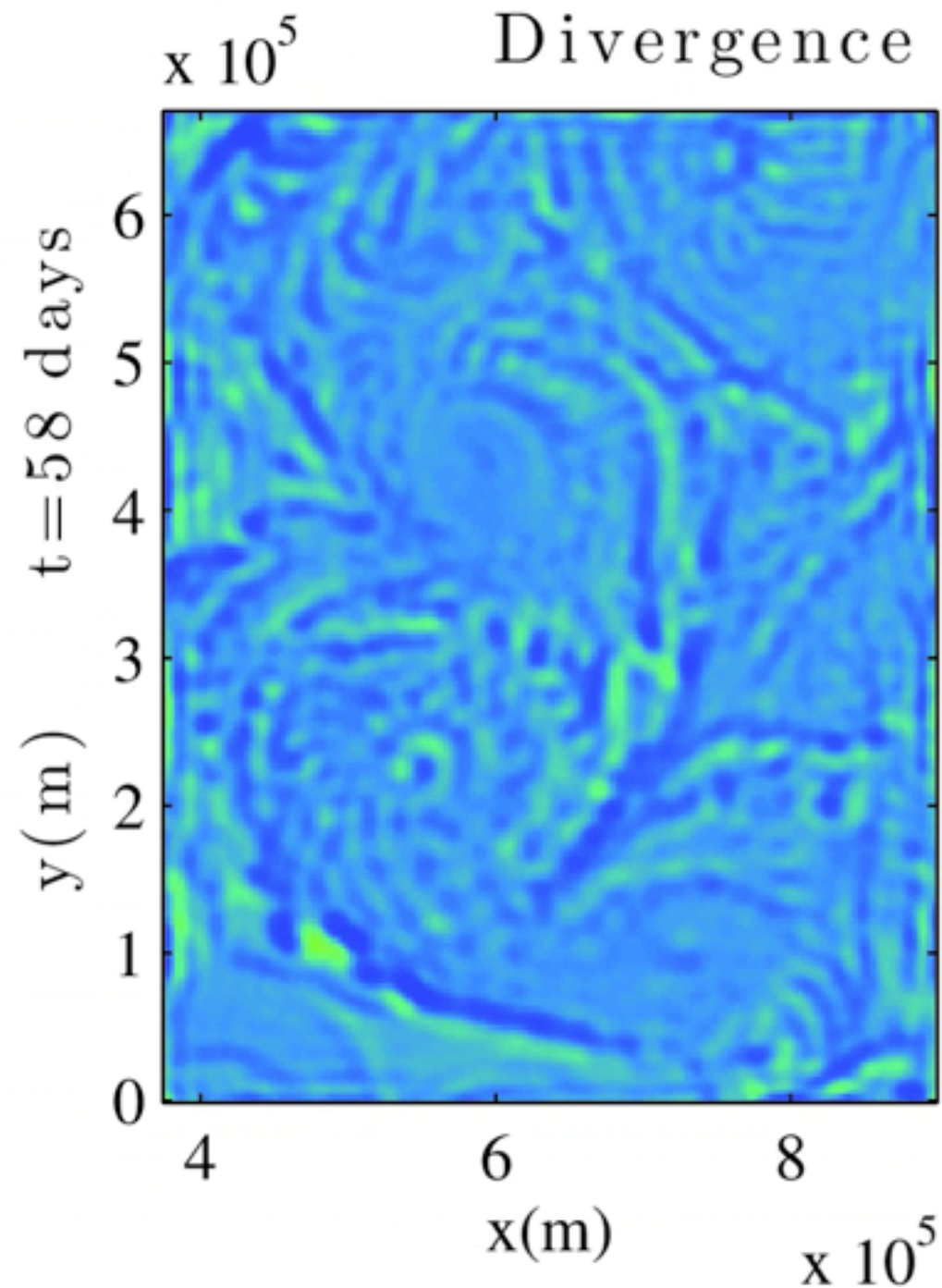


Filtering of model outputs:

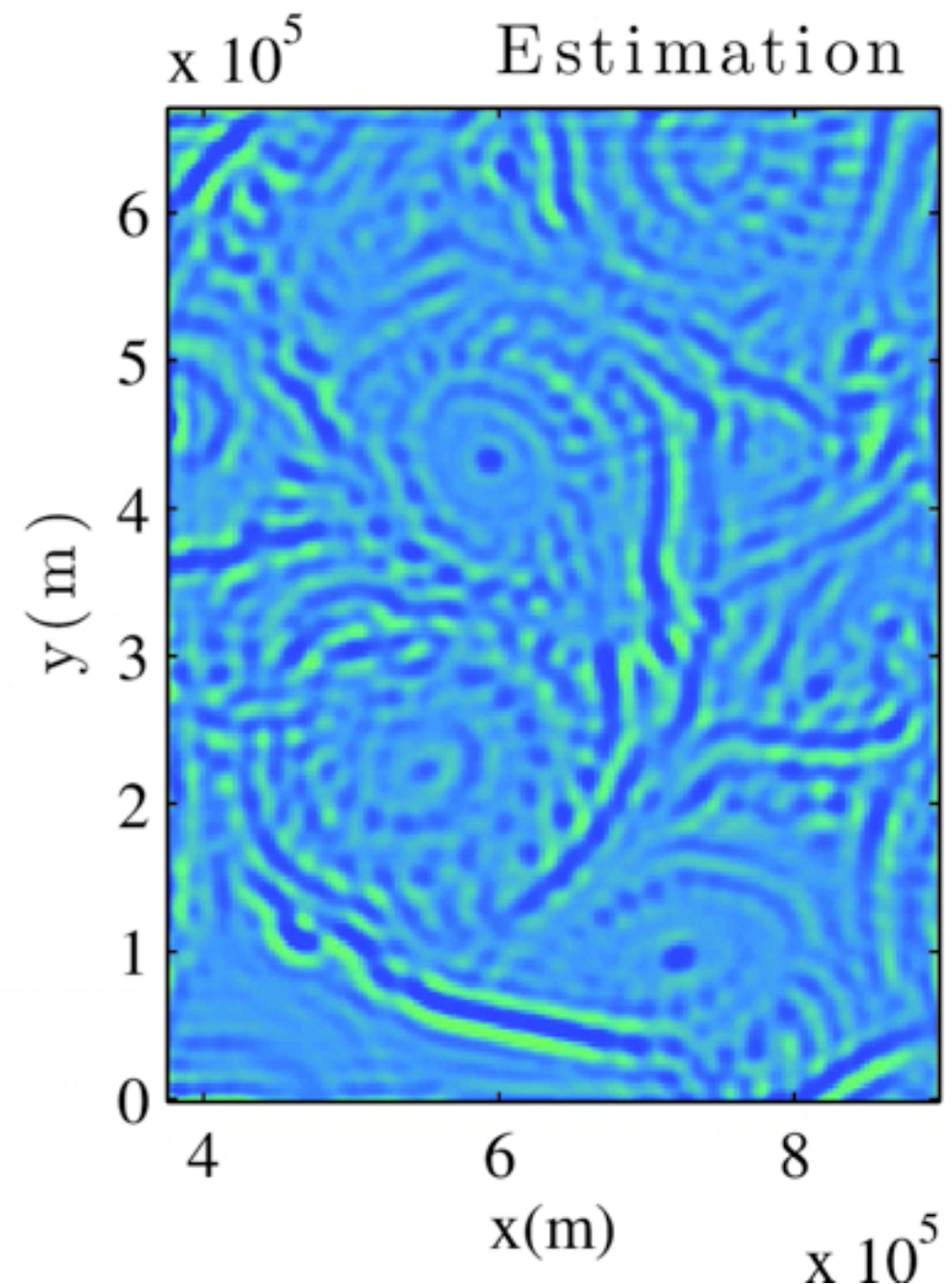
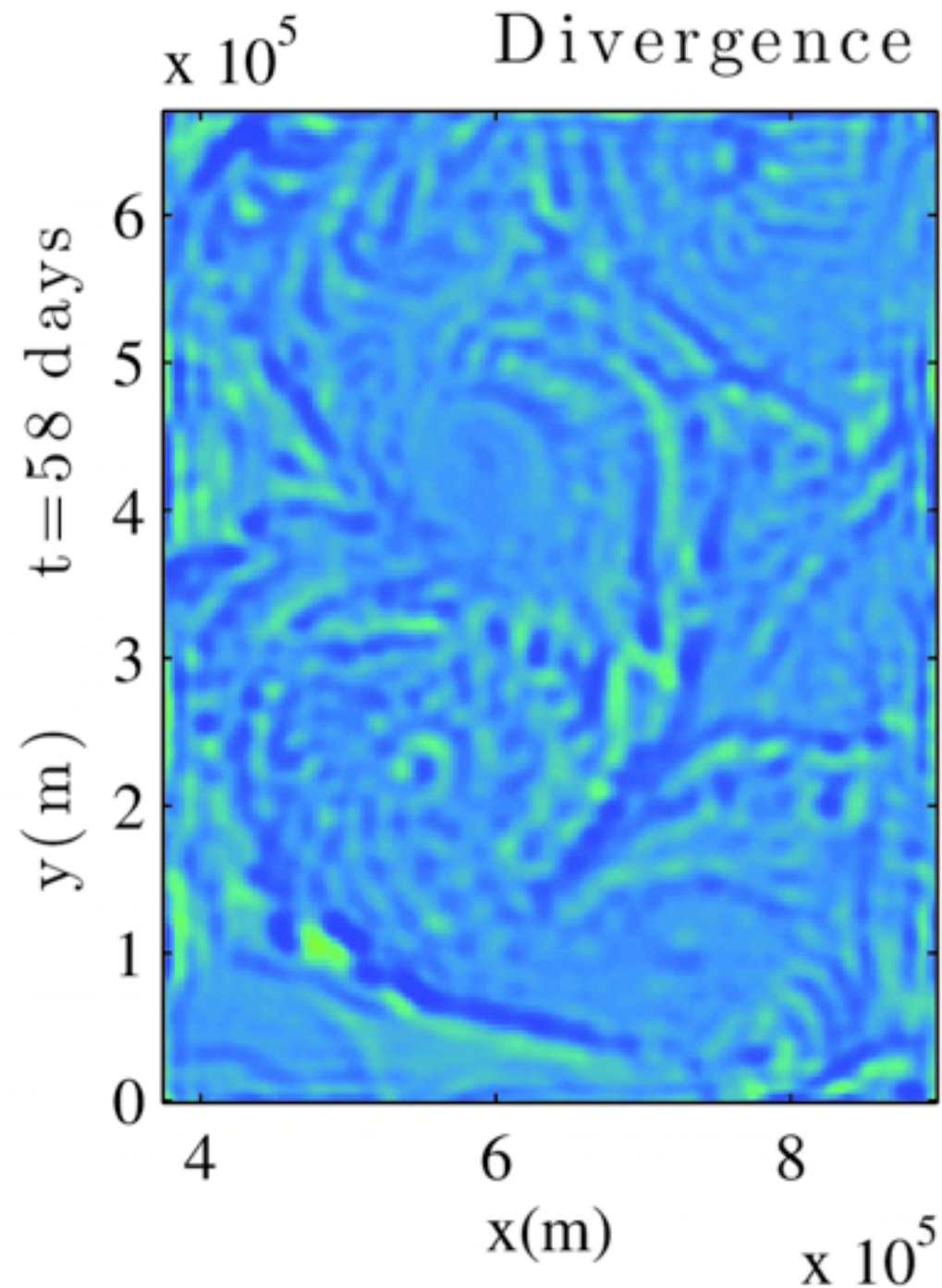
Gula, Jonathan, M. Jeroen Molemaker, and James C. McWilliams
"Gulf Stream dynamics along the southeastern US seaboard."
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Spatial test



Spatial test



Spectral test

